FUNDAMENTALS of ELECTRIC CIRCUITS-SOLUTION MANUAL

AN INFORMATION TO THE STUDENT!

Reference for the solutions is Fundamentals of Electric Circuits-4th Edition Alexander Sadiku.

This book has been prepared to provide elegant and clean solutions for application problems and "problems" sections that are meticulously found at the end of each unit!

The solutions for the "problems" section are prepared for odd numbered questions!

While making the calculations, the significant figures were taken into consideration and the necessary procedures were applied meticulously!

Some problems may not be solved because they are not needed!

In the practice problems and applications sections, just one problem will be solved at the end of each unit!

As we know, some symbols or representations differ in Europe and America. The representations of the referenced book will be applied in this book!

Recommendation!

If you feel inadequate in Linear Algebra, the course that underpins Circuit Theory, you should qualify yourself in Linear Algebra before reviewing this book!

The recommended book for you to understand the basics of Linear Algebra is Linear Algebra: A Modern Introduction by David Poole.

If you see any errors or something that needs improvement, you can inform the e-mail account below.

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Chapter 1-Practice Problems

1.1

As it is known, the charge of one proton is $1.602 \times 10^{-19} C$.

This implies that, $(4 \times 10^{6})(1.602 \times 10^{-19} C) = \mathbf{6.408 \times 10^{-13} C}$

1.2

$i(t) = \frac{dq}{dt} = \frac{d(10-10e^{-2t})}{dt} = 20 \cdot e^{-2t}$

This implies that, $i(0.5) = 7.36 \times 10^{-3} A = \mathbf{7.36\ mA}$

1.3

$i(t) = \frac{dq}{dt} \Rightarrow q(t) = \int_0^t i(\psi)\, d\psi$

This implies that, $\int_0^2 i(\psi)\, d\psi = \int_o^1 i(\psi)\, d\psi + \int_1^2 i(\psi)\, d\psi = \int_0^1 (2A)\, d\psi + \int_1^2 (2\psi^2 A)\, d\psi$

So that, $q = (2\psi)\Big|_{\psi=0}^{\psi=1} + \left(\frac{2}{3}t^3\right)\Big|_{\psi=1}^{\psi=2} = (2) \cdot (1) - (2).(0) + \frac{2}{3}(2^3) - \frac{2}{3}(1^3) = \mathbf{6.667C}$

1.4

It's known that, $v = \frac{\Delta w}{\Delta q}$

This implies that, *(a)* $v_{ab} = \frac{-30J}{2C} = -15V$ *(b)* $v_{ab} = \frac{-30J}{-6C} = \mathbf{5V}$

1.5

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i$

This implies that, *(a)* $p(t) = 2i \cdot (5\cos(60\pi t)) = 50\cos^2(60\pi t) \Rightarrow p(0.005) = 17.27W$

(b) $v = \left(10 + 5\int_0^t (i)\, d\psi\right) = 10 + \left(5\int_0^t 5\cos(60\pi\psi)\, d\psi\right) = 10 + \frac{25}{60\pi}\sin(60\pi t)$

This implies that,

$p(t) = v \cdot i = \left[10 + \frac{25}{60\pi}\sin(60\pi t)\right].[5\cos(60\pi t)] \Rightarrow p(0.005) = \mathbf{29.7W}$

1.6

By using the definitions of power, voltage and current,

$i = \frac{\Delta q}{\Delta t}$ *and* $v = \frac{\Delta w}{\Delta q} \Rightarrow \Delta t = \frac{\Delta q}{i} = \frac{\Delta w}{i \cdot \Delta v}$ *It's obvious that* $t_0 = 0!$

This implies that, $t = \frac{w}{i \cdot v} = \frac{60 \times 10^3 J}{15A \cdot 240v} = \mathbf{16.667s}$

1.7

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i$

Power with respect to voltage drop direction gives, $p = v \cdot i$ *otherwise* $p = -v \cdot i$

$v_3 = 3A \cdot 0.6I$ *and* $I = 5A \Rightarrow v_3 = \mathbf{3V}$

Therefore,

$p_1 = -5V \cdot 8A = \mathbf{-40W}$

$p_2 = 2V \cdot 8A = \mathbf{16W}$

$p_3 = 3V \cdot 3A = \mathbf{9W}$

$p_4 = 3V \cdot 5A = \mathbf{15W}$

So that, negative signed values mean power is supplied, positive signed values mean power is absorbed!

1.8

Let n be the number of electrons and V_0 *as acceleration voltage!*

This implies that, $i = \frac{dq}{dt} = e \cdot \frac{dn}{dt} = e$

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i$

Now it's clear-cut that, power in the beam equals to $p_{beam} = V_0 \cdot i$

So that, $p_{beam} = V_0 \cdot i = (30 \times 10^3 V) \cdot (1.602 \times 10^{-19} C) = 48 \times 10^{-3} W = \mathbf{48mW}$

1.9

Minimum monthly charge = \$12.00, First 100 kWh × \$0.16/kWh = \$16.00

Next 200 kWh × \$0.10/kWh = \$20.00, Remaining charge 100 kWh × \$0.06/kWh = \$6.00

This implies that, average cost = \$54/[100+200+100] = ***13.5 Cents/kWh***

Chapter 1- Problems

1.1

It's known that one electron charge is $e = \mathbf{-1.602 \times 10^{-19}}$

Therefore,

(a) $(6.482 \times 10^{17}) \cdot (-1.602 \times 10^{-19}) = \mathbf{-0.1038C}$

(b) $(1.24 \times 10^{18}) \cdot (-1.602 \times 10^{-19}) = \mathbf{-0.199C}$

(c) $(2.46 \times 10^{17}) \cdot (-1.602 \times 10^{-19}) = \mathbf{-3.94C}$

(d) $(1.628 \times 10^{17}) \cdot (-1.602 \times 10^{-19}) = \mathbf{-26.08C}$

1.3

By the definition of current, $i(t) = \frac{dq}{dt} \Rightarrow q(t) = \int_0^t i(\psi)\, d\psi$

Therefore,

(a) $q(t) = q(0) + \int_0^t i(\psi)\, d\psi = q(0) + \int_0^t (3A)\, d\psi = 1C + 3t|_{\psi=0}^{\psi=t} = (1+3t)C$

(b) $q(t) = q(0) + \int_0^t i(\psi)\, d\psi = q(0) + \int_0^t (2\psi + 5)\, d\psi = q(0) + \psi^2 + 5\psi|_{\psi=0}^{\psi=t} = (t^2 + 5t)C$

(c) $q(t) = q(0) + \int_0^t i(\psi)\, d\psi = q(0) + \int_0^t 20\,cos\left(10\psi + \frac{\pi}{6}\right) d\psi = \left(2\,sin\left(10t + \frac{\pi}{b}\right) + 1\right) \mu C$

(d) $q(t) = q(0) + \int_0^t i(\psi)\, d\psi$

$= q(0) + \int_0^t 10 \cdot e^{-30\psi} \cdot \sin 40\psi\, d\psi = \boldsymbol{-e^{-30t}[0.16\,cos\,40t + 0.12\,sin\,40t]C}$

1.5

By the definition of current, $i(t) = \frac{dq}{dt} \Rightarrow q(t) = \int_0^t i(\psi)\, d\psi$

This implies that, $q = \int_0^{10} i(t)\, dt = \int_0^{10} \left(\frac{1}{2}\psi\right) d\psi = \frac{\psi^2}{4}\Big|_{\psi=0}^{\psi=10} = \boldsymbol{25C}$

1.7

By the definition of current, $i(t) = \frac{dq(t)}{dt}$

This implies that, the slope of the graph gives the current for the time interval!

$25A, \quad 0 < t < 2,$

$-25A, \quad 2 < t < 6,$

$25A, \quad 6 < t < 8.$

So that graph can be drawn by using the data above!

1.9

By the definition of current, $i(t) = \frac{dq}{dt} \Rightarrow q(t) = \int_0^t i(\psi)\, d\psi$

This implies that the area of the region between the line segments and the x-axis gives the charge for the time interval!

So that,

$\int_0^1 i(t)\, dt = (10) \cdot (1) = 10C$ $\quad$ $\int_1^2 i(t)\, dt = (5 + 10) \cdot \left(\frac{1}{2}\right) = 7.5C$

$\int_2^3 i(t)\, dt = (5) \cdot (1) = 5C$ $\quad$ $\int_3^4 i(t)\, dt = (5) \cdot (1) = 5C$

CONT...

$\int_4^5 i(t)\,dt = (5)\cdot\left(\frac{1}{2}\right) = 2.5C$

This implies that,

$10C, \quad 0 < t < 1,$

$7.5C, \quad 1 < t < 2,$

$5C, \quad 2 < t < 3,$

$5C, \quad 3 < t < 4,$

$2.5C, \quad 4 < t < 5.$

So that,

(a) ***10C*** *(b)* ***22.5C*** *(c)* ***30C***

1.11

By the definition of current, $i(t) = \frac{dq}{dt} \Rightarrow q(t) = \int_0^t i(\psi)\,d\psi$

This implies that, $i = \frac{\Delta q}{\Delta t} \Rightarrow \Delta q = i \cdot \Delta t$

So that, $q = i \cdot t = (85 \times 10^{-3}A)\cdot\left(12h \cdot \frac{3600s}{1h}\right) = 3672C = \mathbf{3.672kC}$

By the definition of voltage, $v = \frac{dw(q)}{dq}$

This implies that, $v = \frac{\Delta w}{\Delta q}$

It's obvious that q_0 *and* w_0 *equals to zero!*

So that, $w = v \cdot q = (3672C)\cdot(1.2V) = 4406J = \mathbf{4.406kJ}$

1.13

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq}\cdot\frac{dq(t)}{dt} = v \cdot i$

By the definition of current, $i(t) = \frac{dq}{dt}$

So that, $i(t) = \frac{dq(t)}{dt} = \frac{d(10\sin 4\pi t)}{dt} = 40\pi\cos 4\pi t$ *(Note that this value is in mA!)*

This implies that, $p(t) = i(t)\cdot v(t) = (80\pi\cos^2(4\pi t)) \Rightarrow p(0.3) = 250.2mW$

By the definition of power, $p(t) = \frac{dw(q)}{dt} \Rightarrow w(t) = \int_0^t p(\psi)\,d\psi$

This implies that, $w(0.6) = \int_0^{0.6} p(\psi)\,d\psi = \int_0^{0.6} 80\pi\cos^2(4\pi\psi)\,d\psi$

$= \mathbf{5\,sin\left(\frac{24\pi}{5}\right) + 24\pi = 78.34mJ}$

1.15

By the definition of current, $i(t) = \frac{dq}{dt} \Rightarrow q(t) = \int_0^t i(\psi)\, d\psi$

This implies that, $q(t) = \int_0^t i(\psi)\, d\psi \Rightarrow q(2) = \int_0^2 3 \cdot e^{-2t}\, d\psi = 3\left(\frac{1}{2} - \frac{e^{-4}}{2}\right) = 1.473C$

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i$

Also, $v(t) = 5\frac{di}{dt} = \frac{5\, d(3e^{-2t})}{dt} = 5\big((-2) \cdot (3e^{-2t})\big) = -30e^{-2t}$

This implies that, $p(t) = v(t) \cdot i(t) = (-30e^{-2t}) \cdot (3e^{-2t}) = -90e^{-4t}W$

By the definition of power, $p(t) = \frac{dw(q)}{dt} \Rightarrow w(t) = \int_0^t p(\psi)\, d\psi$

This implies, $w(t) = \int_0^t p(\psi)\, d\psi = \int_0^3 -90e^{-4\psi}\, d\psi = -90\left(\frac{1}{4} - \frac{e^{-12}}{4}\right) = \mathbf{-22.5J}$

1.17

As it known, $\sum p = p_{absorbed} + p_{delivered} = \mathbf{0}!$

This implies that, $p_1 + p_2 + p_3 + p_4 + p_5 = 0 \Rightarrow p_3 = -(P_1 + p_2 + p_4 + p_5) = -(-70W) = \mathbf{70W}$

1.19

As it known, $\sum p = p_{absorbed} + p_{delivered} = \mathbf{0}!$

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i$

Note that voltage drop directions are signed positive!

Therefore,

$\sum p = -(4) \cdot (9) + (1) \cdot (9) + (I) \cdot (3) + (I) \cdot (6) = 0 \Rightarrow 9I = 27 \Rightarrow I = \mathbf{3A}$

1.21

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i$

Also we've known that, $i(t) = \frac{dq}{dt} \Rightarrow i = \frac{\Delta q}{\Delta t}$

It's obvious that, t_0 *and* q_0 *equals to zero!*

Rearranging the equations gives,

$$q = i \cdot t = \frac{p}{v} \cdot t = \frac{60w}{120v} \cdot \left(24h \cdot \frac{36005}{1h}\right) = 43200C = \mathbf{43.2kC}$$

Let denote n_e *as number of electrons!*

As it known, $n_e = \left|\frac{q}{e}\right| = \left|\frac{43200}{-1.602\times10^{-19}}\right| = \mathbf{2.697 \times 10^{23}}$

END!

Chapter 2-Practice Problems

2.1

By the Ohm's Law, $v = i \cdot R$

This implies that, $v = i \cdot R \Rightarrow i = \frac{v}{R} = \frac{110v}{10\Omega} = \mathbf{11A}$

2.2

It's obvious that, given current i equals to the current that supplied by the independent current source!

By the Ohm's Law, $v = i \cdot R$

This implies that, $v = i \cdot R = (2 \times 10^{-3}A) \cdot (10 \times 10^{3}\Omega) = \mathbf{20V}$

As it known, $R = \frac{1}{G} \Rightarrow G = \frac{1}{R} = \frac{1}{10 \times 10^{3}\Omega} = 10^{-4}S = 100\mu S$

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i$

Note that voltage drop directions are signed positive!

So that, $p = v \cdot i = 20V \cdot (2 \times 10^{-3}A) = 40 \times 10^{-3}W = \mathbf{40mW}$

2.3

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i \Rightarrow i = \frac{p}{v}$

This implies that, $i = \frac{p}{v} = \frac{20\cos^2 t}{10\cos t} = (2\cos t)mA$

By the Ohm's Law, $v = i \cdot R \Rightarrow \ R = \frac{v}{i}$

This implies, $R = \frac{v}{i} = \frac{10\cos t}{2\cos t \times 10^{-3}} = 5 \times 10^{3}\Omega = \mathbf{5k\Omega}$

2.4

By the definition of branch, number of single elements gives the number of branches!

So that, 4 branches and 1 independent voltage source gives, $4 + 1 = \mathbf{5}$*!*

By the definition of node, number of connections between branches gives the number of nodes!

This implies that there exist 3 different nodes!

2.5

Let denote KCL as Kirchhoff's Current Law and KVL as Kirchhoff's Voltage Law!

Assume that direction of voltage drops signed positive and direction of current i through the loop is clockwise direction!

So by KVL, $-10 + v_1 - 8 - v_2 = \mathbf{0}$

By Ohm's Law, $v = i \cdot R \Rightarrow v_1 = 4i$ *and* $v_2 = -2i$

CONT . . .

This implies, $v_1 - v_2 = 4i - (-2i) = 18 \Rightarrow i = 3A$ *so,* $v_1 = 4i = 4 \cdot 3 = 12V$

$v_2 = -2i = (-2) \cdot (3) = \mathbf{-6V}$

2.6

Let denote KCL as Kirchhoff's Current Law and KVL as Kirchhoff's Voltage Law!

Assume that direction of voltage drops signed positive and direction of current i through the loop is clockwise direction!

So by KVL, $-35 + v_x + 2v_x - v_0 = 0$

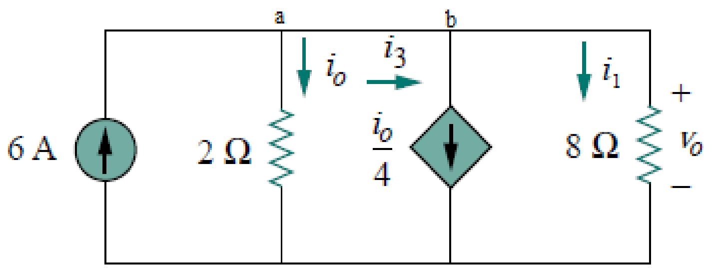
By Ohm's Law, $v = i \cdot R \Rightarrow v_x = 10i$ *and* $v_0 = -5i$

This implies that, $3v_x - v_0 = 3 \cdot (10i) - (-5i) = 35 \Rightarrow i = 1A$

So that, $v_x = 10i = 10 \cdot 1 = 10V$ *and* $v_0 = -5i = (-5) \cdot (1) = \mathbf{-5V}$

2.7

Let redraw the circuit with nodes and important currents!



Let denote currents leaving a node positive!

By KCL for node a and node b gives, $-6 + i_0 + i_3 = 0$ *and* $-i_3 + \frac{i_0}{4} + i_1 = 0$

Assume that direction of voltage drops signed positive!

It's obvious that, $v_{2\Omega} = v_0$ *because they are connected in same pair of terminals!*

By Ohm's Law, $v = i \cdot R \Rightarrow v_{2\Omega} = 2i_0$ *and* $v_0 = 8i_1$

This implies that, $2i_0 = 8i_1 \Rightarrow i_0 = 4i_1$*!*

By substitution, $6 - i_0 = \frac{i_0}{4} + i_1 \Rightarrow 6 - i_0 = \frac{i_0}{4} + \frac{i_0}{4} \Rightarrow i_0 = 4A$

As it known, $v_0 = 8i_1 = 8\left(\frac{i_0}{4}\right) = 2i_0 = \mathbf{8V}$

2.8

Let denote currents leaving a node positive!

By KCL for upper middle node, $-i_1 + i_2 + i_3 = \mathbf{0}$

Assume that direction of voltage drops signed positive!

CONT . . .

By KVL for both left and right closed paths, $-5 + v_1 + v_2 = \mathbf{0}$ *and* $-v_2 + v_3 - 3 = \mathbf{0}$

Ohm's Law gives, $v_1 = 2i_1, v_2 = 8i_2$ *and* $v_3 = 4i_3$

By substitution, $2i_1 + 8i_2 = \mathbf{5}$ *and* $-8i_2 + 4i_3 = \mathbf{3}$

Now we have 3 unknowns and 3 equations!

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A^{-1}Ax} = \boldsymbol{A^{-1}b} \Rightarrow \boldsymbol{x} = \boldsymbol{A^{-1}b}$

$$\begin{bmatrix} -1 & 1 & 1 \\ 2 & 8 & 0 \\ 0 & -8 & 4 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0.25 \\ 1.25 \end{bmatrix}$$

This implies that, $i_1 = 1.5A, i_2 = 0.25A$ *and* $i_3 = 1.25A$

So that, $v_1 = 2i_1 = 2 \cdot (1.5A) = \mathbf{3V},\ v_2 = 8i_2 = 8 \cdot (0.25A) = \mathbf{2V},\ v_3 = 4i_3 = 4 \cdot (1.25A) = \mathbf{5V}$

2.9

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

It's obvious that, 4Ω *and* 5Ω *are connected in series so* R_{eq} *for them is* $4\Omega + 5\Omega = 9\Omega$

Now, 9Ω *and* 3Ω *are connected in series so* R_{eq} *for them is* $9\Omega + 3\Omega = 12\Omega$

It's clear cut that, 9Ω *and* 3Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{12} + \frac{1}{4} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 3\Omega$

Now, 3Ω *and* 3Ω *are connected in series so* R_{eq} *for them is* $3\Omega + 3\Omega = 6\Omega$

It's clear- cut that, 6Ω *and* 6Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{6} + \frac{1}{6} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 3\Omega$

It's obvious that, , $2\Omega, 3\Omega$ *and* 1Ω *are connected in series so* R_{eq} *for them is* $2\Omega + 3\Omega + 1\Omega = \mathbf{6\Omega}$

This implies, the equivalent resistance of the circuit is $\mathbf{6\Omega}$

2.10

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

It's clear that, 20Ω *and* 5Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{20} + \frac{1}{5} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 4\Omega$

Now, 4Ω *and* 1Ω *are connected in series so* R_{eq} *for them is* $4\Omega + 1\Omega = 5\Omega$

It's clear that, 5Ω *and* 20Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{20} + \frac{1}{5} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 4\Omega$

CONT...

Now, 4Ω *and* 2Ω *are connected in series so* R_{eq} *for them is* $4\Omega + 2\Omega = \mathbf{6\Omega}$

It's clear that, 6Ω *and* 9Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{6} + \frac{1}{9} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = \mathbf{3.6\Omega}$

Now, 3.6Ω *and* 18Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{18} + \frac{1}{3.6} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = \mathbf{3\Omega}$

Finally, 3Ω *and* 8Ω *are connected in series so* R_{eq} *for them is* $3\Omega + 8\Omega = \mathbf{11\Omega}$

2.11

Let denote equivalent resistance as R_{eq} *and equivalent counductance as* G_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

This implies that, $G_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{G_n}\right)^{-1}$ *for the series connections and* $G_{eq} = \sum_{n=1}^{\infty} G_n$*!*

It's obvious that, $8S$ *and* $4S$ *resistors are connected parallel so* G_{eq} *for them is* $8S + 4S = 12S$

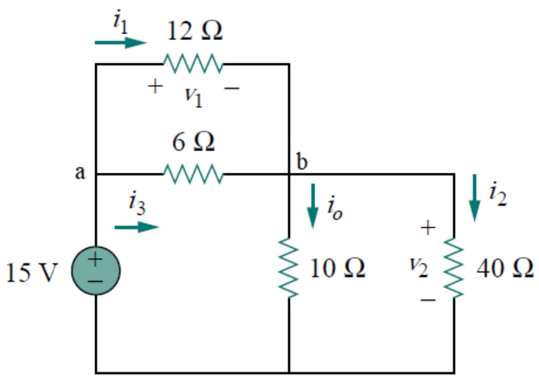
Now, , $6S$ *and* $12S$ *resistors are connected in series so* G_{eq} *for them is* $\frac{1}{6} + \frac{1}{12} = \frac{1}{G_e} \Rightarrow G_e = 4S$

It's obvious that, $4S$ *and* $2S$ *resistors are connected parallel so* G_{eq} *for them is* $4S + 2S = 6S$

Now, $6S$ *and* $12S$ *resistors are connected in series so* G_{eq} *for them is* $\frac{1}{6} + \frac{1}{12} = \frac{1}{G_e} \Rightarrow G_e = \mathbf{4S}$

2.12

Let redraw the circuit with nodes and important currents!



Note that Voltage and Current Division will make things easier but the basic operations are shown to make everything clear!

Let denote currents leaving a node positive!

So by KCL for node b gives, $-i_3 - i_1 + i_0 + i_2 = 0$

CONT...

It's obvious that, $v_1 = v_{6\Omega}$ *and* $v_2 = v_{40\Omega}$ *because they are both connected to the same pair of terminals!(if there is any confusion you can verify it using Kirchhoff's Voltage Law!)*

By Ohm's Law, $v = i \cdot R \Rightarrow v_1 = v_{6\Omega} = 12i_1 = 6i_3$ *and* $v_2 = v_{10\Omega} = 40i_2 = 10i_0$

Assume that direction of voltage drops signed positive!

By KVL for left closed path with clockwise direction gives, $-15 + v_{6\Omega} + v_{10\Omega} = 0$

By substitution, $12i_1 + 40i_2 = \mathbf{15V}$

So rearranging the equations gives,

$-i_1 + i_2 - i_3 + i_0 = \mathbf{0}$

$12i_1 + 0i_2 - 6i_3 + 0i_0 = 0$

$0i_1 + 40i_2 + 0i_3 - 10i_0 = 0$

$12i_1 + 40i_2 + 0i_3 + 0i_0 = 15$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} -1 & 1 & -1 & 1 \\ 12 & 0 & -6 & 0 \\ 0 & 40 & 0 & -10 \\ 12 & 40 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 15 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_0 \end{bmatrix} = \begin{bmatrix} 0.4167 \\ 0.25 \\ 0.8333 \\ 1 \end{bmatrix}$$

This implies that, $i_1 = 416.7mA, i_2 = 250mA, i_3 = 833.3mA$ *and* $i_0 = 1A$

Now by substitution, $v_1 = v_{6\Omega} = 12i_1 = 5V,\ v_2 = v_{10\Omega} = 40i_2 = 10V$

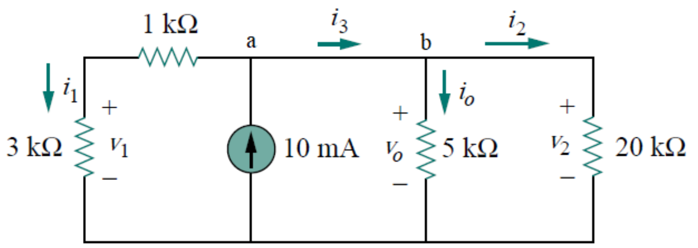
By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i$

Note that direction of voltage drops signed positive!

This implies, $p_1 = v_1 \cdot i_1 = 2083mW = \mathbf{2.083W}$ *and* $p_2 = v_2 \cdot i_2 = 2500mW = \mathbf{2.5W}$

2.13

Let redraw the circuit with nodes and important currents!



Note that Voltage and Current Division will make things easier but the basic operations are shown to make everything clear!

CONT...

Let denote currents leaving a node positive!

By KCL for both nodes a and b gives, $i_1 - 0.01 + i_3 = 0$ *and* $-i_3 + i_0 + i_2 = 0$

It's obvious that, $v_0 = v_2 = v_{10mA}$ *because they are connected to the same pair of terminals!(if there is any confusion you can verify it using Kirchhoff's Voltage Law!)*

By Ohm's Law, $v = i \cdot R \Rightarrow v_0 = v_2 = 5000i_0 = 20000i_2, v_{1k\Omega} = 1000i_1$ *and* $v_1 = 3000i_1$

Assume that direction of voltage drops signed positive!

By KCL for both leftmost closed path gives, $-v_1 - v_{1k\Omega} + v_0 = 0$

By substitution, $-3000i_1 - 1000i_1 + 5000i_0 = \mathbf{0}$

So rearranging the equations gives

$i_1 + 0i_2 + i_3 + 0i_0 = 0.01$

$0i_1 + i_2 - i_3 + i_0 = 0$

$0i_1 - 20000i_2 + 0i_3 + 5000i_0 = 0$

$-4000i_1 + 0i_2 + 0i_3 + 5000i_0 = 0$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & -20000 & 0 & 5000 \\ -4000 & 0 & 0 & 5000 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_0 \end{bmatrix} = \begin{bmatrix} 0.01 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_0 \end{bmatrix} = \begin{bmatrix} 5 \times 10^{-3} \\ 1 \times 10^{-3} \\ 5 \times 10^{-3} \\ 4 \times 10^{-3} \end{bmatrix}$$

This implies that, $i_1 = 5mA, i_2 = 1mA, i_3 = 5mA$ *and* $i_0 = 4mA$

Now by substitution, $v_0 = v_2 = 5000i_0 = 20V$ *and* $v_1 = 3000i_1 = 15V$

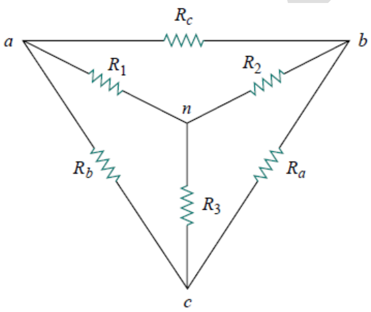
By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i$

Note that direction of voltage drops signed positive!

This implies, $p_{3k\Omega} = v_1 \cdot i_1 = 75 \times 10^{-3}W = 75mW$ *and* $p_{20k\Omega} = v_{2 \cdot i_2} = 20 \times 10^{-3}W = \mathbf{20mW}$

2.14

Let redraw the circuit with the Δ *and* Y *form nested! (This form is called Superposition of* Δ *and* Y *networks!)*



CONT…

By setting $R_{ac}(Y) = R_{ac}(\Delta)$, $R_{ab}(Y) = R_{ab}(\Delta)$ *and* $R_{bc}(Y) = R_{bc}(\Delta)$,

$R_{ac} = R_1 + R_3 = \frac{R_b(Ra+R_c)}{Ra+R_b+R_c}$, $R_{ab} = R_1 + R_2 = \frac{R_c(R_a+R_b)}{R_a+R_b+Rc}$ *and* $R_{bc} = R_2 + R_3 = \frac{R_a(R_b+R_c)}{Ra+R_b+Rc}$

By subtraction and additions, $R_1 = \frac{R_bR_c}{R_a+R_b+R_c}$, $R_2 = \frac{R_cR_a}{R_a+R_b+Rc}$ *and* $R_3 = \frac{R_aR_b}{R_a+R_b+R_c}$

By using multiplication and division,

$R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1}$, $R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2}$ *and* $R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3}$

Now, $R_1R_2 + R_2R_3 + R_3R_1 = (10)\cdot(20) + (20)\cdot(40) + (40)\cdot(10) = 1400$

This implies that,

$R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1} = \frac{1400}{10} = \mathbf{140\Omega}$,

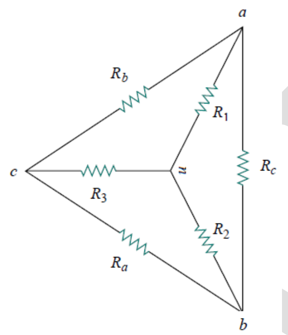
$R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2} = \frac{1400}{20} = \mathbf{70\Omega}$,

$R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3} = \frac{1400}{40} = \mathbf{35\Omega}$

2.15

Let redraw the part of the circuit with Superposition that will be converted from wye to delta!

Let denote, 20Ω *as* R_3, 50Ω *as* R_2 *and* 10Ω *as* R_1!



By setting $R_{ac}(Y) = R_{ac}(\Delta)$, $R_{ab}(Y) = R_{ab}(\Delta)$ *and* $R_{bc}(Y) = R_{bc}(\Delta)$,

$R_{ac} = R_1 + R_3 = \frac{R_b(Ra+R_c)}{Ra+R_b+R_c}$, $R_{ab} = R_1 + R_2 = \frac{R_c(R_a+R_b)}{R_a+R_b+Rc}$ *and* $R_{bc} = R_2 + R_3 = \frac{R_a(R_b+R_c)}{Ra+R_b+Rc}$

By subtraction and additions, $R_1 = \frac{R_bR_c}{R_a+R_b+R_c}$, $R_2 = \frac{R_cR_a}{R_a+R_b+Rc}$ *and* $R_3 = \frac{R_aR_b}{R_a+R_b+R_c}$

By using multiplication and division,

$R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1}$, $R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2}$ *and* $R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3}$

Now, $R_1R_2 + R_2R_3 + R_3R_1 = (10)\cdot(50) + (50)\cdot(20) + (20)\cdot(10) = 1700$

This implies that,

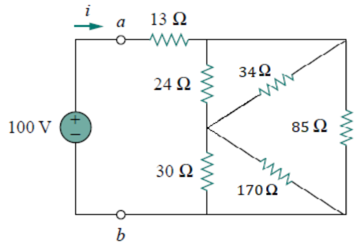
$R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1} = \frac{1700}{10} = \mathbf{170\Omega}$,

CONT...

$R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2} = \frac{1700}{50} = \mathbf{34\Omega},$

$R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3} = \frac{1700}{20} = \mathbf{85\Omega}$

So that the new circuit becomes,



Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

It's clear that, 24Ω *and* 34Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{24} + \frac{1}{34} = \frac{1}{R_{eq}}$

$\Rightarrow R_{eq} = \mathbf{14.07\Omega}$

It's obvious that , 30Ω *and* 170Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{30} + \frac{1}{170} = \frac{1}{R_{eq}}$

$\Rightarrow R_{eq} = \mathbf{25.5\Omega}$

Now, 14.07Ω *and* 25.5Ω *are connected in series so* R_{eq} *for them is* $14.07\Omega + 25.5\Omega = 39.6\Omega$

It's clear that, 85Ω *and* 39.6Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{85} + \frac{1}{39.6} = \frac{1}{R_{eq}}$

$\Rightarrow R_{eq} = \mathbf{27\Omega}$

Finally, 27Ω *and* 13Ω *are connected in series so* R_{eq} *for them is* $27\Omega + 13\Omega = 40\Omega$

This implies, the equivalent resistance of the circuit is 40Ω*!*

Assume that direction of voltage drops signed positive!

By KVL for closed path in clockwise direction gives, $-100V + v_{40\Omega} = 0 \Rightarrow v_{40\Omega} = 100V$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R} = \frac{100}{40} = \mathbf{2.5A}$

2.16

(a) By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i$

Power with respect to voltage drop direction gives, $p = v \cdot i$ *otherwise* $p = -v \cdot i$

As it known, $\sum p = p_{absorbed} + p_{delivered} = 0$

CONT...

Let denote total power absorbed by bulbs as p_{bulbs}*!*

This implies that, $p_{bulbs} - p_{110v} = 0 \Rightarrow p_{110v} = p_{bulbs}$

So that, $p_{110v} = P_b = (40) \cdot (10) = 400W$

Now, $p = v \cdot i \Rightarrow i = \frac{p}{v} \Rightarrow\ i_{110v} = \frac{400}{110} = 3.64A$

Now, the source current must be divided equally into 10 parts. As a result, an equal amount of current must pass through each bulb!

This implies,the current for each bulb "i_b*" is,* $i_b = \frac{i_{110v}}{10} = 0.364A$

(b) By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i$

Power with respect to voltage drop direction gives, $p = v \cdot i$ *otherwise* $p = -v \cdot i$

As it known, $\sum p = p_{absorbed} + p_{delivered} = 0$

Let denote total power absorbed by bulbs as p_{bulbs}*!*

This implies that, $p_{bulbs} - p_{110v} = 0 \Rightarrow p_{110v} = p_{bulbs}$

So that, $p_{110v} = P_b = (40) \cdot (10) = 400W$

Now, $p = v \cdot i \Rightarrow i = \frac{p}{v} \Rightarrow\ i_{110v} = \frac{400}{110} = \mathbf{3.64A}$

Because of series connection, each bulb will be supplied by the same amount of current which corresponds "source current"!

Chapter 2- Problems

2.3

By the definition of resistance, $R = p\frac{l}{A}$ *where* p *denotes resistivity,* l *denotes length and* A *denotes cross-sectional area!*

This implies that, $R = p\frac{l}{A} = p\frac{l}{\pi r^2} \Rightarrow r = \sqrt{\frac{R}{p} \cdot \frac{\pi}{l}}$

So that, $r = \sqrt{\frac{R}{p} \cdot \frac{\pi}{l}} = \mathbf{184.3mm}$

2.11

Assume that direction of voltage drops signed positive!

By KVL for both left and right closed paths with the clockwise direction gives,

$-v_1 + 1 + 5 = 0 \Rightarrow v_1 = \mathbf{6V}$ *and* $-5 + 2 + v_2 = 0 \Rightarrow v_2 = \mathbf{3V}$

2.13

Let denote currents leaving a node positive!

By KCL for middle first, middle second, middle third and middle last gives,

$-I_1 - I_2 + 2 = 0,\qquad I_2 + 3 + 7 = 0,$

$-7 + I_3 - I_4 = 0,\qquad -2 + I_4 + 4 = 0$

It's obvious that, $I_2 = \mathbf{-10A}$ *and* $I_4 = \mathbf{-2A}$*, also by using substitution,* $I_1 = \mathbf{12A}$ *and* $I_3 = \mathbf{5A}$

2.15

Assume that direction of voltage drops signed positive!

By KVL for both left and right closed paths with the clockwise direction gives,

$-12 + v + 2 = 0 \Rightarrow 10V$ *and* $-2 + 8 + 3i_x = 0 \Rightarrow i_x = \mathbf{-2A}$

2.17

Assume that direction of voltage drops signed positive!

By KVL for both upper and lower triangular closed paths with the clockwise directions gives,

$-24 + v_1 - v_2 = 0$ *and* $v_2 + v_3 + 12 = 0$

It's obvious that, $v_3 = 10V$ *because they are connected to the same pair of terminals!(if there is any confusion you can verify it using Kirchhoff's Voltage Law!)*

By substitution, $v_2 = \mathbf{-22V}$ *and* $v_1 = \mathbf{2V}$

2.19

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i$

Source power with respect to passive sign convention gives, $p_{source} = -v_{source} \cdot i_{source}$

This implies that,

$p_{20V} = (-20) \cdot (I) = -20I,$

$p_{10V} = -(-10) \cdot (I) = 10I,$

$p_{-4V} = -(-4) \cdot (I) = 4I$

Assume that direction of voltage drops signed positive!

By KVL for closed path with the clockwise direction gives,

$-20 + 10 + v_{3\Omega} - (-4) = 0 \Rightarrow v_{3\Omega} = 6V$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R} \Rightarrow I = \frac{6}{3} = 2A$

This implies, $p_{20V} = \mathbf{-40W}, p_{10V} = 20W, p_{-4V} = \mathbf{8W}$

As it known, $\sum p = p_{absorbed} + p_{delivered} = 0$

This implies, $-40 + 20 + 8 + p_3 = 0 \Rightarrow p_3 = \mathbf{12W}$

2.21

Assume that direction of voltage drops signed positive!

By KVL for closed path with the clockwise direction gives,

$-15 + v_{1\Omega} + 2v_x + v_x + v_{2\Omega} = 0$

By Ohm's Law, $v = i \cdot R$

This implies that, $v_{1\Omega} = i_{source}$, $V_{2\Omega} = 2i_{source}$ and $V_x = 5i_{source}$

By substitution, $v_x = \frac{15-(v_{1\Omega}+v_{2\Omega})}{3} \Rightarrow V_x = 5i_{source} = \frac{15-(i_{source}+2i_{source})}{3} \Rightarrow i_{source} = 0.833A$

This implies, $V_x = 5i_{source} = \mathbf{4.17V}$

2.23

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

It's clear that, 3Ω *and* 6Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{3} + \frac{1}{6} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 2\Omega$

Now, 2Ω *and* 4Ω *are connected in series so* R_{eq} *for them is* $2\Omega + 4\Omega = 6\Omega$

It's clear that, 8Ω *and* 12Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{8} + \frac{1}{12} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 4.8\Omega$

Now, 1.2Ω *and* 4.8Ω *are connected in series so* R_{eq} *for them is* $1.2\Omega + 4.8\Omega = 6\Omega$

It's clear that, 6Ω *and* 6Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{6} + \frac{1}{6} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 3\Omega$

Finally, 1Ω *and* 3Ω *are connected in series so* R_{eq} *for them is* $1\Omega + 3\Omega = 4\Omega$

Let denote the current passing through the v_x *as* i_x*!*

Now by Current Division equation, $i_x = \frac{R_{eq}}{R_x} i_{source}$

Here R_{eq} *corresponds the equivalent resistance of the circuit which can be calculated as,*

$\frac{1}{2} + \frac{1}{4} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 1.3\Omega$

Also R_x *corresponds the resistance that is combined parallel with* 2Ω*,that is* 4Ω*!*

So by substitution, $i_x = \frac{R_e}{R_x} i_{source} = \frac{1.3}{4} 6A = 2.0A$

By Ohm's Law, $v = i \cdot R$

This implies that, $v_x = i_x \cdot (1) = 2V$

Now let apply the same process to 2 rightmost closed paths, the current i_{source} *passing through the* 1.2Ω *will be* $1A$*!*

CONT . . .

By Current Division equation, $i_x = \frac{R_{eq}}{R_x} i_{source}$

Here R_{eq} *corresponds the equivalent resistance of the circuit which can be calculated as,*

$\frac{1}{8} + \frac{1}{12} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 4.8\Omega$

Also R_x *corresponds the resistance that is combined parallel with* 8Ω*, that is* 12Ω*!*

So by substitution, $i_x = \frac{R_{eq}}{R_x} i_{source} = \frac{4.8}{12} 1A = 0.4A$

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i = i^2 \cdot R$

This implies, $p_{12\Omega} = (0.4)^2 \cdot (12) = \mathbf{1.92W}$

2.25

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

Here R_{eq} *corresponds the equivalent resistance of the middle and the rightmost closed paths! which can be calculated as,*

$\frac{1}{5} + \frac{1}{20} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = \mathbf{4k\Omega}$

Now, Current Division equation is applied for the part of the circuit that covers the middle and the rightmost closed paths!

$$i_{20k\Omega} = \frac{4}{20} 0.01V_0$$

It's obvious that, $v_{5mA} = V_0$ *because they are connected to same pair of terminals!(if there is any confusion you can verify it using Kirchhoff's Voltage Law!)*

By Ohm's Law, $v = i \cdot R \Rightarrow V_0 = v_{5mA} = (5 \times 10^{-3}) \cdot (10 \times 10^3) = 50V$

By substitution,

$V_0 = 50V \Rightarrow i_{20k\Omega} = 0.1A,$

$v_{20k\Omega} = (0.1) \cdot (20 \times 10^3) = 2000V = 2kV,$

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i$

This implies that, $p_{20k\Omega} = v_{20k\Omega} \cdot i_{20k\Omega} = (i_{20k\Omega})^2 \cdot R_{20k\Omega}$

Now it's clear that, $p_{20k\Omega} = (2000) \cdot (0.1) = 200W = \mathbf{0.2kW}$

2.27

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections of resistors!*

CONT...

It's obvious that, 4Ω *and* 6Ω *are connected in series so* R_{eq} *for them is* $4\Omega + 6\Omega = 10\Omega$

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

By substitution, $v_0 = \frac{4}{10} 16V = \mathbf{6.4V}$

2.29

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

Let apply the process from the rightmost to leftmost resistance!

It's clear that, 1Ω *and* 1Ω *are connected in series so* R_{eq} *for them is* $1\Omega + 1\Omega = 2\Omega$

It's obvious that, 2Ω *and* 1Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{2} + \frac{1}{1} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 0.667\Omega$

Now, 0.667Ω *and* 1Ω *are connected in series so* R_{eq} *for them is* $0.667\Omega + 1\Omega = 1.667\Omega$

It's obvious that, 1.667Ω *and* 1Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{1.667} + \frac{1}{1} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 0.625\Omega$

It's clear that, 1Ω *and* 0.625Ω *are connected in series so* R_{eq} *for them is* $1\Omega + 0.625\Omega = \mathbf{1.625\Omega}$

2.31

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

It's obvious that, 2Ω*,* 1Ω *and* 4Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{2} + \frac{1}{1} + \frac{1}{4} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 0.571\Omega$

It's clear that, 3Ω *and* 0.571Ω *are connected in series so* R_{eq} *for them is*

$R_{eq} = 3\Omega + 0.571\Omega = 3.571\Omega$

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

By substitution, $v_{3\Omega} = \frac{3}{3.571} 40V = 33.6V$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

CONT...

This implies, $i_1 = \frac{33.6}{3} = 11.2A$

As it known, , 2Ω, 1Ω *and* 4Ω *are connected in parallel so* R_{eq} *for them is* $R_{eq} = 0.571\Omega$*!*

By Current Division equation, $i_x = \frac{R_{eq}}{R_x} i_{source}$

Here i_{source} *corresponds the source current which is equal to,* $i_1 = 11.2A$

This implies,

$i_2 = \frac{0.571}{4} 11.2A = 1.60A$

$i_4 = \frac{0.571}{1} 11.2A = 6.40A$

$i_5 = \frac{0.571}{2} 11.2A = 3.20A$

Let denote currents leaving a node positive!

By KCL for the node that connects 1Ω *and* 2Ω *gives,*

$-i_3 + i_4 + i_5 = 0 \Rightarrow i_3 = i_4 + i_5 = \mathbf{9.60A}$

2.33

Let denote equivalent resistance as R_{eq} *and equivalent conductance as* G_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

This implies that, $G_{eq} = \sum_{n=1}^{\infty} G_n$ *for the parallel connections and* $G_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{G_n}\right)^{-1}$ *for series connections of resistors!*

It's obvious that, $6S$ *and* $3S$ *are connected in series so so* G_{eq} *for them is* $\frac{1}{6} + \frac{1}{3} = \frac{1}{G_{eq}} \Rightarrow G_{eq} = 2S$

It's clear that, $2S$ *and* $2S$ *are connected in parallel so* G_{eq} *for them is* $2S + 2S = 4S$

Now, $4S$ *and* $4S$ *are connected in series so* G_{eq} *for them is* $\frac{1}{4} + \frac{1}{4} = \frac{1}{G_{eq}} \Rightarrow G_{eq} = 2S$

Finally, $2S$ *and* $1S$ *are connected in parallel so* G_{eq} *for them is* $2S + 1S = 3S$

By Current Division equation, $i_x = \frac{R_{eq}}{R_x} i_{source} \Rightarrow \frac{G_x}{G_{eq}} i_{source}$

This implies, $i = \frac{2}{3} 9A = 6A$

Let denote currents leaving a node positive and i_1 *as the current passing through the* $3S$ *resistor!*

By KCL for the node that connects $4S$ *and* $1S$ *gives,* $-9 + i + i_1 = 0 \Rightarrow i_1 = 9 - i = 3A$

By Ohm's Law, $v = i \cdot R \Rightarrow v = \frac{i}{G} = \frac{3}{1} = 3V$

CONT...

2.35

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

It's obvious that, 70Ω *and* 30Ω *are connected in parallel so* R_{eq} *for them is*

$$\frac{1}{70} + \frac{1}{30} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 21\Omega$$

Now, 20Ω *and* 5Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{20} + \frac{1}{5} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 4\Omega$

It's obvious that, $v_{70\Omega} = v_{30\Omega}$ *and* $v_{20\Omega} = V_0$ *because they are connected to same pair of terminals!(if there is any confusion you can verify it using Kirchhoff's Voltage Law!)*

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

This implies, $v_{30\Omega} = v_{70\Omega} = \frac{21}{25} 50 = 42V$ *and* $v_{20\Omega} = V_0 = \frac{4}{25} 50 = 8V$

Let denote i_1 *as current passing through the* 30Ω *and* i_2 *as current passing through the* 5Ω *resistor!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

Now, $v_{30\Omega} = 30 i_1 \Rightarrow i_1 = \mathbf{1.4A}$ *and* $v_0 = 5 i_2 \Rightarrow i_2 = \mathbf{1.6A}$

Let denote currents leaving a node positive!

By KCL for rightmost middle node gives, $-I_0 - i_1 + i_2 = 0 \Rightarrow I_0 = i_2 - i_1 = \mathbf{0.2A}$

2.37

Let denote the current that flows around the circuit as i_s*!*

Assume that direction of voltage drops signed positive!

By KVL for closed path with the clockwise direction gives,

$$-20 + 10 + v_{10\Omega} - 30 = 0 \Rightarrow v_{10\Omega} = 40V$$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

This implies, $v_{10\Omega} = 10 i_s \Rightarrow i_s = 4A$

Also, $v_R = i_s \cdot R \Rightarrow R = \mathbf{2.5\Omega}$

2.39

(a) Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

CONT...

It's obvious that, $2k\Omega$ *and* $1k\Omega$ *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{2}+\frac{1}{1}=\frac{1}{R_{eq}} \Rightarrow R_{eq} = \mathbf{0.667k\Omega}$

Now, $0.667k\Omega$ *and* $2k\Omega$ *are connected in series so* R_{eq} *for them is* $0.667k\Omega + 2k\Omega = 2.667\Omega$

It's clear that, $2.667k\Omega$ *and* $1k\Omega$ *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{2.667}+\frac{1}{1}=\frac{1}{R_{eq}} \Rightarrow R_{eq} = 0.7273k\Omega = 727.3\Omega$

(b) It's clear that, $12k\Omega$ *and* $12k\Omega$ *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{12}+\frac{1}{12}=\frac{1}{R_{eq}} \Rightarrow R_{eq} = 6k\Omega$

Now, $6k\Omega$ *and* $6k\Omega$ *are connected in series so* R_{eq} *for them is* $6k\Omega + 6k\Omega = 12k\Omega$

Finally, $12k\Omega$ *and* $4k\Omega$ *are connected in parallel so* R_{eq} *for them is* $\frac{1}{12}+\frac{1}{4}=\frac{1}{R_{eq}} \Rightarrow R_{eq} = \mathbf{3k\Omega}$

2.41

Let denote equivalent resistance as R_{eq} *!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

It's obvious that, 12Ω*,* 12Ω *and* 12Ω *are connected in parallel so* R_{eq} *for them is*

$$\frac{1}{12}+\frac{1}{12}+\frac{1}{12}=\frac{1}{R_{eq}} \Rightarrow R_{eq} = 4\Omega$$

Now, 4Ω *and* R *are connected in series so* R_{eq} *for them is* $4\Omega + R$ *!*

It's clear that, 30Ω *and* 60Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{30}+\frac{1}{60}=\frac{1}{R_{eq}} \Rightarrow R_{eq} = 20\Omega$

Now, 10Ω *and* 20Ω *are connected in series so* R_{eq} *for them is* $10\Omega + 20\Omega = 30\Omega$

Finally, $4\Omega + R$ *and* 30Ω *are connected in series so* R_{eq} *for them is*

$4\Omega + R + 30\Omega = 50\Omega \Rightarrow R = 16\Omega$

2.43

(a) Let denote equivalent resistance as R_{eq} *!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

CONT...

It's clear that, 10Ω *and* 40Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{10}+\frac{1}{40}=\frac{1}{R_{eq}} \Rightarrow R_{eq}=8\Omega$

Now, 5Ω *and* 20Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{5}+\frac{1}{20}=\frac{1}{R_{eq}} \Rightarrow R_{eq}=4\Omega$

Finally, 8Ω *and* 4Ω *are connected in series so* R_{eq} *for them is* $8\Omega+4\Omega=\mathbf{12\Omega}$

(b) It's clear that, $60\Omega, 20\Omega$ *and* 30Ω *are connected in parallel so* R_{eq} *for them is*

$$\frac{1}{60}+\frac{1}{20}+\frac{1}{30}=\frac{1}{R_{eq}} \Rightarrow R_{eq}=10\Omega$$

Now, 10Ω *and* 10Ω *are connected in series so* R_{eq} *for them is* $10\Omega+10\Omega=\mathbf{20\Omega}$

Finally, 80Ω *and* 20Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{80}+\frac{1}{20}=\frac{1}{R_{eq}} \Rightarrow R_{eq}=\mathbf{16\Omega}$

2.45

(a) Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq}=\sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq}=\left(\sum_{n=1}^{\infty}\frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

It's obvious that, 50Ω *and* 5Ω *are connected in series so* R_{eq} *for them is* $50\Omega+5\Omega=55\Omega$

It's clear that, $10\Omega, 40\Omega, 20\Omega$ *and* 30Ω *are connected in parallel so* R_{eq} *for them is*

$$\frac{1}{10}+\frac{1}{40}+\frac{1}{20}+\frac{1}{30}=\frac{1}{R_{eq}} \Rightarrow R_{eq}=\mathbf{4.8\Omega}$$

It's clear that, 55Ω *and* 4.8Ω *are connected in series so* R_{eq} *for them is* $55\Omega+4.8\Omega=59.8\Omega$

(b) It's clear that, 12Ω *and* 60Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{12}+\frac{1}{60}=\frac{1}{R_{eq}} \Rightarrow R_{eq}=10\Omega$

Now, 10Ω *and* 20Ω *are connected in series so* R_{eq} *for them is* $10\Omega+20\Omega=30\Omega$

It's obvious that, 30Ω *and* 30Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{30}+\frac{1}{30}=\frac{1}{R_{eq}} \Rightarrow R_{eq}=\mathbf{15\Omega}$

Now, 15Ω *and* 10Ω *are connected in series so* R_{eq} *for them is* $15\Omega+10\Omega=25\Omega$

It's clear that, 25Ω *and* 25Ω *are connected in parallel so* R_{eq} *for them is*

CONT...

$\frac{1}{25} + \frac{1}{25} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 12.5\Omega$

Finally, $5\Omega, 12.5\Omega$ *and* 15Ω *are connected in series so* R_{eq} *for them is*

$5\Omega + 12.5\Omega + 15\Omega = 32.5\Omega$

2.47

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

It's obvious that, 5Ω *and* 20Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{5} + \frac{1}{20} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 4\Omega$

Likewise, 6Ω *and* 3Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{6} + \frac{1}{3} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 2\Omega$

Finally, $10\Omega, 4\Omega$ *,* 2Ω *and* 8Ω *are connected in series so* R_{eq} *for them is*

$10\Omega + 4\Omega + 2\Omega + 8\Omega = \mathbf{24\Omega}$

2.49

(a) By setting $R_{ac}(Y) = R_{ac}(\Delta)$*,* $R_{ab}(Y) = R_{ab}(\Delta)$ *and* $R_{bc}(Y) = R_{bc}(\Delta)$*,*

$R_{ac} = R_1 + R_3 = \frac{R_b(Ra+R_c)}{Ra+R_b+R_c}$, $R_{ab} = R_1 + R_2 = \frac{R_c(R_a+R_b)}{R_a+R_b+Rc}$ *and* $R_{bc} = R_2 + R_3 = \frac{R_a(R_b+R_c)}{Ra+R_b+Rc}$

By subtraction and additions, $R_1 = \frac{R_bR_c}{R_a+R_b+R_c}$, $R_2 = \frac{R_cR_a}{R_a+R_b+Rc}$ *and* $R_3 = \frac{R_aR_b}{R_a+R_b+R_c}$

Therefore,

$R_1 = \frac{R_bR_c}{R_a+R_b+R_c} = \frac{(12)\cdot(12)}{12+12+12} = \mathbf{4\Omega}$

$R_2 = \frac{R_cR_a}{R_a+R_b+Rc} = \frac{(12)\cdot(12)}{12+12+12} = \mathbf{4\Omega}$

$R_3 = \frac{R_aR_b}{R_a+R_b+R_c} = \frac{(12)\cdot(12)}{12+12+12} = \mathbf{4\Omega}$

(b) By applying the same process,

$R_1 = \frac{R_bR_c}{R_a+R_b+R_c} = \frac{(30)\cdot(60)}{10+30+60} = \mathbf{18\Omega}$

$R_2 = \frac{R_cR_a}{R_a+R_b+Rc} = \frac{(60)\cdot(10)}{10+30+60} = \mathbf{6\Omega}$

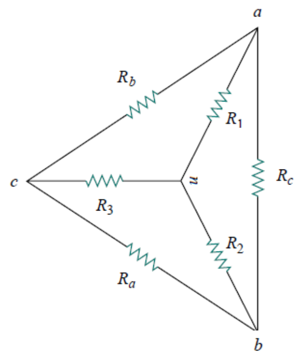
$R_3 = \frac{R_aR_b}{R_a+R_b+R_c} = \frac{(10)\cdot(30)}{10+30+60} = \mathbf{3\Omega}$

2.51

Let redraw the part of the circuit with Superposition that will be converted from wye to delta!

CONT...

Let denote, 10Ω *as* R_3, 20Ω *as* R_2 *and* 20Ω *as* R_1*!*



By setting $R_{ac}(Y) = R_{ac}(\Delta)$, $R_{ab}(Y) = R_{ab}(\Delta)$ *and* $R_{bc}(Y) = R_{bc}(\Delta)$,

$R_{ac} = R_1 + R_3 = \frac{R_b(Ra+R_c)}{Ra+R_b+R_c}$, $R_{ab} = R_1 + R_2 = \frac{R_c(R_a+R_b)}{R_a+R_b+Rc}$ *and* $R_{bc} = R_2 + R_3 = \frac{R_a(R_b+R_c)}{Ra+R_b+Rc}$

By subtraction and additions, $R_1 = \frac{R_bR_c}{R_a+R_b+R_c}$, $R_2 = \frac{R_cR_a}{R_a+R_b+Rc}$ *and* $R_3 = \frac{R_aR_b}{R_a+R_b+R_c}$

By using multiplication and division,

$R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1}$, $R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2}$ *and* $R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3}$

Now, $R_1R_2 + R_2R_3 + R_3R_1 = (20)\cdot(20) + (20)\cdot(10) + (10)\cdot(20) = \mathbf{800}$

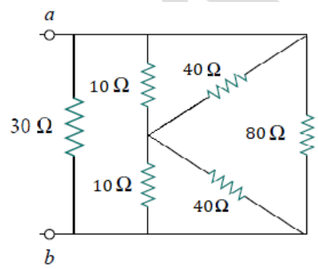
This implies that,

$R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1} = \frac{800}{20} = 40\Omega,$

$R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2} = \frac{800}{20} = 40\Omega,$

$R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3} = \frac{800}{10} = 80\Omega$

So that the new circuit becomes,



Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

It's obvious that, 10Ω *and* 40Ω *are connected in parallel so* R_{eq} *for them is*

CONT...

$\frac{1}{10}+\frac{1}{40}=\frac{1}{R_{eq}} \Rightarrow R_{eq}=\mathbf{8\Omega}$

Likewise, 10Ω *and* 40Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{10}+\frac{1}{40}=\frac{1}{R_{eq}} \Rightarrow R_{eq}=8\Omega$

It's obvious that, 8Ω *and* 8Ω *are connected in series so* R_{eq} *for them is*

$8\Omega + 8\Omega = 16\Omega$

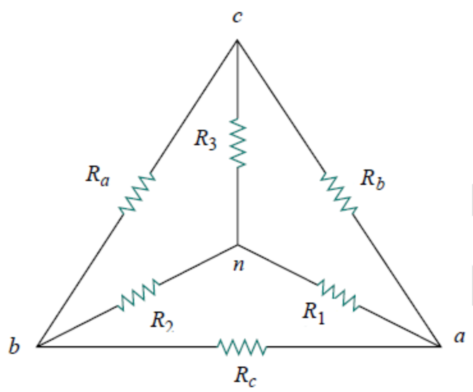
It's clear that, $30\Omega, 16\Omega$ *and* 80Ω *are connected in parallel so* R_{eq} *for them is*

$$\frac{1}{30}+\frac{1}{16}+\frac{1}{80}=\frac{1}{R_{eq}} \Rightarrow R_{eq}=\mathbf{9.2\Omega}$$

2.53

Let redraw the part of the circuit with Superposition that will be converted from wye to delta!

Let denote, 10Ω *as* R_3*,* 60Ω *as* R_2 *and* 50Ω *as* R_1*!*



$R_{ac}=R_1+R_3=\frac{R_b(Ra+R_c)}{Ra+R_b+R_c}$, $R_{ab}=R_1+R_2=\frac{R_c(R_a+R_b)}{R_a+R_b+Rc}$ and $R_{bc}=R_2+R_3=\frac{R_a(R_b+R_c)}{Ra+R_b+Rc}$

By subtraction and additions, $R_1=\frac{R_bR_c}{R_a+R_b+R_c}$, $R_2=\frac{R_cR_a}{R_a+R_b+Rc}$ *and* $R_3=\frac{R_aR_b}{R_a+R_b+R_c}$

By using multiplication and division,

$R_a=\frac{R_1R_2+R_2R_3+R_3R_1}{R_1}$, $R_b=\frac{R_1R_2+R_2R_3+R_3R_1}{R_2}$ *and* $R_c=\frac{R_1R_2+R_2R_3+R_3R_1}{R_3}$

Now, $R_1R_2+R_2R_3+R_3R_1=(50)\cdot(60)+(60)\cdot(10)+(10)\cdot(50)=4100$

This implies that,

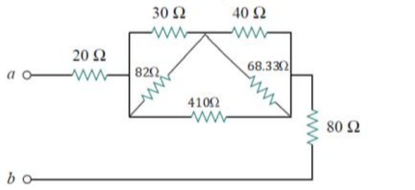
$R_a=\frac{R_1R_2+R_2R_3+R_3R_1}{R_1}=\frac{4100}{50}=82\Omega,$

$R_b=\frac{R_1R_2+R_2R_3+R_3R_1}{R_2}=\frac{4100}{60}=68.33\Omega,$

$R_c=\frac{R_1R_2+R_2R_3+R_3R_1}{R_3}=\frac{4100}{10}=410\Omega$

So that the new circuit becomes,

CONT...



Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

It's obvious that, 68.33Ω *and* 40Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{68.33} + \frac{1}{40} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 25.23\Omega$

Likewise, 30Ω *and* 82Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{30} + \frac{1}{82} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 21.96\Omega$

It's obvious that, 25.23Ω *and* 21.96Ω *are connected in series so* R_{eq} *for them is*

$25.23\Omega + 21.96\Omega = 47.19\Omega$

It's obvious that, 47.19Ω *and* 410Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{47.19\Omega} + \frac{1}{410} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 42.32\Omega$

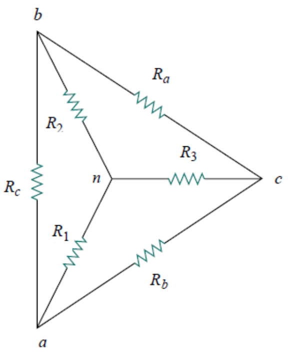
Finally, 20Ω*,* 42.32Ω *and* 80Ω *are connected in series so* R_{eq} *for them is*

$20\Omega + 42.32\Omega + 80\Omega = \mathbf{142.32\Omega}$

2.55

Let redraw the part of the circuit with Superposition that will be converted from wye to delta!

Let denote, 40Ω *as* R_3*,* 20Ω *as* R_2 *and* 10Ω *as* R_1*!*



By setting $R_{ac}(Y) = R_{ac}(\Delta)$*,* $R_{ab}(Y) = R_{ab}(\Delta)$ *and* $R_{bc}(Y) = R_{bc}(\Delta)$*,*

$R_{ac} = R_1 + R_3 = \frac{R_b(Ra+R_c)}{Ra+R_b+R_c}$*,* $R_{ab} = R_1 + R_2 = \frac{R_c(R_a+R_b)}{R_a+R_b+Rc}$ *and* $R_{bc} = R_2 + R_3 = \frac{R_a(R_b+R_c)}{Ra+R_b+Rc}$

CONT...

By subtraction and additions, $R_1 = \frac{R_b R_c}{R_a+R_b+R_c}$, $R_2 = \frac{R_c R_a}{R_a+R_b+Rc}$ *and* $R_3 = \frac{R_a R_b}{R_a+R_b+R_c}$

By using multiplication and division,

$R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1}$, $R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2}$ *and* $R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3}$

Now, $R_1R_2 + R_2R_3 + R_3R_1 = (10)\cdot(20) + (20)\cdot(40) + (40)\cdot(10) = 1400$

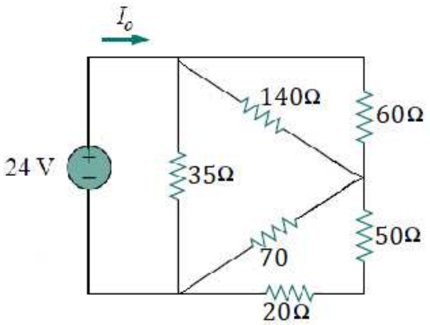
This implies that,

$R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1} = \frac{1400}{10} = 140\Omega,$

$R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2} = \frac{1400}{20} = 70\Omega,$

$R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3} = \frac{1400}{40} = 35\Omega$

So that the new circuit becomes,



Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

It's obvious that, 60Ω *and* 140Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{60} + \frac{1}{140} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 42\Omega$

Now, 20Ω *and* 50Ω *are connected in series so* R_{eq} *for them is* $20\Omega + 50\Omega = 70\Omega$

Likewise, 70Ω *and* 70Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{70} + \frac{1}{70} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 35\Omega$

It's obvious that, 42Ω *and* 35*are connected in series so* R_{eq} *for them is*

$42\Omega + 35\Omega = 77\Omega$

Finally, , 77Ω *and* 35Ω *are connected in parallel so* R_{eq} *for them is*

$\frac{1}{77\Omega} + \frac{1}{35} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 24.0625\Omega$

CONT . . .

It's obvious that 24V= $v_{R_{eq}}$ *because they are connected to same pair of terminals!(if there is any confusion you can verify it using Kirchhoff's Voltage Law!)*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

This implies that, $I_0 = \frac{24}{24.0625} = 0.9974A = 997.4mA$

Chapter 3-Practice Problems

3.1

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for both a and b node gives, $-1 + \frac{v_1}{2} + \frac{v_1 - v_2}{6} = 0$ *and* $\frac{v_2 - v_1}{6} + \frac{v_2}{7} + 4 = 0$

Rearranging the equations gives,

$v_1\left(\frac{1}{2} + \frac{1}{6}\right) + v_2\left(-\frac{1}{6}\right) = 1,\ v_1\left(-\frac{1}{6}\right) + v_2\left(\frac{1}{6} + \frac{1}{7}\right) = -4$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} \frac{4}{6} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{13}{42} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -2 \\ -14 \end{bmatrix}$$

This implies that, $v_1 = \boldsymbol{-2V}$ *and* $v_2 = \boldsymbol{-14V}$

3.2

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node 1, 2 and 3 gives,

$-10 + \frac{v_1 - v_3}{2} + \frac{v_1 - v_2}{3} = 0,$

$\frac{v_2 - v_1}{3} + \frac{v_2}{4} - 4i_x = 0$

$\frac{v_3 - v_1}{2} + 4i_x + \frac{v_3}{6} = 0$

It's obvious that, $i_x = \frac{\boldsymbol{v_2}}{\boldsymbol{4}}$*!*

CONT...

Now we have 4 equations and 4 unknowns!

Rearranging the equations gives,

$$v_1\left(\frac{1}{2}+\frac{1}{3}\right)+v_2\left(-\frac{1}{3}\right)+v_3\left(-\frac{1}{2}\right)+0i_x=10$$

$$v_1\left(-\frac{1}{3}\right)+v_2\left(\frac{1}{3}+\frac{1}{4}\right)+0v_3+i_x(-4)=0$$

$$v_1\left(-\frac{1}{2}\right)+0v_2+v_3\left(\frac{1}{2}+\frac{1}{6}\right)+i_x(4)=0$$

$$0v_1+v_2\left(-\frac{1}{4}\right)+0v_3+i_x=0$$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow\boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow\boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix}\frac{5}{6} & -\frac{1}{3} & -\frac{1}{2} & 0\\ -\frac{1}{3} & \frac{7}{12} & 0 & -4\\ -\frac{1}{2} & 0 & \frac{4}{6} & 4\\ 0 & -\frac{1}{4} & 0 & 1\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\\i_x\end{bmatrix}=\begin{bmatrix}10\\0\\0\\0\end{bmatrix}\Rightarrow\begin{bmatrix}v_1\\v_2\\v_3\\i_x\end{bmatrix}=\begin{bmatrix}80\\-64\\156\\-16\end{bmatrix}$$

This implies that, $v_1=80V$, $v_2=\boldsymbol{-64V}$ *and* $v_3=\boldsymbol{156V}$

3.3

Let denote the lower essential node as reference node!

It's obvious that $9V$ *source connects two non-reference nodes!*

This implies that, $9V$ *source is a supernode!*

By Ohm's Law, $v=i\cdot R\Rightarrow i=\frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i=\frac{v_{higher}-v_{lower}}{R}$

Let denote currents leaving a node positive and v_i *as voltage across the terminals of* 2Ω *resistor!*

By KCL for supernode gives, $\frac{v-21}{4}+\frac{v}{3}+\frac{v_i}{2}+\frac{v_i}{6}=0$

Also it's clear that, $v_i-v=9$

Rearranging the equations gives,

$$v\left(\frac{1}{4}+\frac{1}{3}\right)+v_i\left(\frac{1}{2}+\frac{1}{6}\right)=\frac{21}{4}$$

$$v(-1)+v_i(1)=9$$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow\boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow\boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

CONT . . .

$$\begin{bmatrix} \frac{7}{12} & \frac{4}{6} \\ -1 & 1 \end{bmatrix} \begin{bmatrix} v \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{21}{4} \\ 9 \end{bmatrix} \Rightarrow \begin{bmatrix} v \\ v_i \end{bmatrix} = \begin{bmatrix} -0.6 \\ 8.4 \end{bmatrix}$$

This implies that, $v = \mathbf{-0.6V}$ *and* $v_i = \mathbf{8.4V}$

Using Ohm's Law, $v_i = 2i \Rightarrow i = \frac{v_i}{2} = \mathbf{4.2A}$

3.4

It's obvious that $10V$ *independent voltage and* $5i$ *dependent voltage source connects two non-reference nodes!*

This implies that, $10V$ *and* $5i$ *sources create supernodes!(Note that we can combine them as single larger supernode!)*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for larger supernode gives, $\frac{v_1}{2} + \frac{v_2}{4} + \frac{v_3}{3} = 0$

It's clear that, $v_1 - v_2 = 10$, $v_3 - v_2 = 5i$ *and* $i = \frac{v_1}{2}$*!*

Rearraning the equations gives,

$$v_1\left(\frac{1}{2}\right) + v_2\left(\frac{1}{4}\right) + v_3\left(\frac{1}{3}\right) = 0$$

$$v_1 - v_2 + 0v_3 = 10$$

$$0v_1 + v_3 - v_2 = 5\frac{v_1}{2} \Rightarrow v_1\left(\frac{-5}{2}\right) - v_2 + v_3 = 0$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A^{-1}Ax} = \boldsymbol{A^{-1}b} \Rightarrow \boldsymbol{x} = \boldsymbol{A^{-1}b}$

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{3} \\ 1 & -1 & 0 \\ -\frac{5}{2} & -1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 3.043 \\ -6.956 \\ 0.6521 \end{bmatrix}$$

This implies that, $v_1 = \mathbf{3.043V}$, $v_2 = \mathbf{-6.956V}$ *and* $v_3 = \mathbf{0.6522V}$

3.5

Assume that direction of voltage drops signed positive!

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

By KVL for each mesh gives,

CONT...

$-36 + 2i_1 + 12(i_1 - i_2) + 4i_1 = 0$

$9i_2 + 24 + 3i_2 + 12(i_2 - i_1) = 0$

Rearranging the equations gives,

$18i_1 - 12i_2 = 36$

$-12i_1 + 24i_2 = -24$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 18 & -12 \\ -12 & 24 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 36 \\ -24 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

This implies that, $i_1 = \boldsymbol{2A}, i_2 = \boldsymbol{0A}$

3.6

Assume that direction of voltage drops signed positive!

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

By KVL for each mesh gives,

$-20 + 4(i_1 - i_3) + 2(i_1 - i_2) = 0$

$8(i_2 - i_3) - 10i_3 + 2(i_2 - i_1) = 0$

$6i_3 + 8(i_3 - i_2) + 4(i_3 - i_1) = 0$

Also It's obvious that, $I_0 = i_3$

By substitution, second equation becomes,

Rearranging the equations gives,

$6i_1 - 2i_2 - 4i_3 = 20, -2i_1 + 10i_2 - 18i_3 = 0$ *and* $-4i_1 - 8i_2 + 18i_3 = 0$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 6 & -2 & -4 \\ -2 & 10 & -18 \\ -4 & -8 & 18 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -3.214 \\ -9.642 \\ -5 \end{bmatrix}$$

This implies that, $i_3 = I_0 = \boldsymbol{-5A}$

3.7

Assume that direction of voltage drops signed positive!

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

It's obvious that, independent current source causes a supermesh!(Two meshes have a current source in common!)

By KVL for supermesh gives, $-6 + 2(i_1 - i_3) + 4(i_2 - i_3) + 8i_2 = 0$

By KVL for mesh i_3*gives,* $2i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0$

CONT...

Also it's obvious that, $i_1 - i_2 = 3$

Rearranging the equations gives,

$2i_1 + 12i_2 - 6i_3 = 6,\ -2i_1 - 4i_2 + 8i_3 = 0$ *and* $i_1 - i_2 + 0i_3 = 0$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 2 & 12 & -6 \\ -2 & -4 & 8 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3.474 \\ 0.4737 \\ 1.1052 \end{bmatrix}$$

This implies that, $i_1 = \mathbf{3.474A}$, $i_2 = \mathbf{0.4737A}$ *and* $i_3 = \mathbf{1.1052A}$

3.8

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives,

$$v_1 - v_3 + \frac{v_1 - v_2}{5} + \frac{v_1}{10} = 0$$

$$\frac{v_2 - v_1}{5} - 1 - 2 = 0$$

$$v_3 - v_1 + 1 + \frac{v_3 - v_4}{4} = 0$$

$$\frac{v_4 - v_3}{4} + \frac{v_4}{2} - 3 = 0$$

Rearranging the equations gives,

$$v_1\left(1 + \frac{1}{5} + \frac{1}{10}\right) + v_2\left(-\frac{1}{5}\right) - v_3 + 0v_4 = 0$$

$$v_1\left(-\frac{1}{5}\right) + v_2\left(\frac{1}{5}\right) + 0v_3 + 0v_4 = 3$$

$$-v_1 + 0v_2 + v_3\left(1 + \frac{1}{4}\right) + v_4\left(-\frac{1}{4}\right) = -1$$

$$0v_1 + 0v_2 + v_3\left(-\frac{1}{4}\right) + v_4\left(\frac{1}{2} + \frac{1}{4}\right) = 3$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 1.3 & -0.2 & -1 & 0 \\ -0.2 & 0.2 & 0 & 0 \\ -1 & 0 & 1.25 & -0.25 \\ 0 & 0 & -0.25 & 0.75 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{3} \\ \mathbf{-1} \\ \mathbf{3} \end{bmatrix}$$

So that, node voltages can be found by solving the matrix above!

3.9

Assume that direction of voltage drops signed positive!

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

By KVL for each mesh gives,

$-24 + 50i_1 + 40(i_1 - i_2) + 80(i_1 - i_4) = 0$

$30(i_2 - i_3) + 10(i_2 - i_4) + 40(i_2 - i_1) = 0$

$12 + 20(i_3 - i_5) + 30(i_3 - i_2) = 0$

$-10 + 80(i_4 - i_1) + 10(i_4 - i_3) = 0$

$20(i_5 - i_3) + 60i_5 + 10 = 0$

Rearranging the equations gives,

$170i_1 - 40i_2 + 0i_3 - 80i_4 + 0i_5 = 24$

$-40i_1 + 80i_2 - 30i_3 - 10i_4 + 0i_5 = 0$

$0i_1 - 30i_2 + 50i_3 + 0i_4 - 20i_5 = -12$

$-80i_1 - 10i_2 + 0i_3 + 90i_4 + 0i_5 = 10$

$0i_1 + 0i_2 - 20i_3 + 0i_4 + 80i_5 = -10$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 170 & -40 & 0 & -80 & 0 \\ -40 & 80 & -30 & -10 & 0 \\ 0 & -30 & 50 & 0 & -20 \\ -80 & -10 & 0 & 90 & 0 \\ 0 & 0 & -20 & 0 & 80 \end{bmatrix} \begin{bmatrix} i_1 \\ i_1 \\ i_1 \\ i_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ -12 \\ 10 \\ -10 \end{bmatrix}$$

So that, mesh currents can be found by solving the matrix above!

3.10

Note that PSpice must be used but in order to make everything clear, regular process will be applied!

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives, $\frac{v_1}{30} + \frac{v_1}{60} + 2 = 0$ *and* $-2 + \frac{v_2}{50} + \frac{v_2}{25} + \frac{v_2 - v_3}{100} = 0$

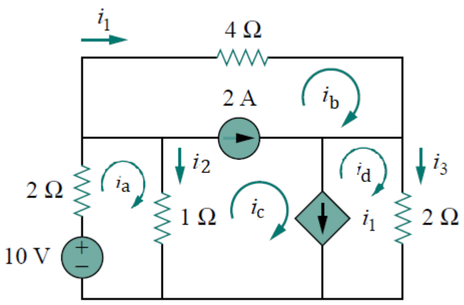
And also it's clear that, $v_3 = 200V$

This implies that, $v_1 = -40V$ *and* $v_2 = 57.14V$

CONT . . .

3.11

Let redraw the circuit with mesh currents!



It's obvious that independent current source and dependent current source are in common with different meshes,

So that they're both results in supermeshes! (Note that we can think them as single large supermesh!)

Assume that direction of voltage drops signed positive!

By KVL for large supermesh gives, $4i_b + 2i_d + i_c - i_a = 0$

By KVL for mesh i_a *gives,* $-10 + 2i_a + i_a - i_c = 0$

It's clear that, $i_c - i_b = 2$

We can clearly see that, $i_1 = i_b \Rightarrow i_c - i_d = i_1 = i_b \Rightarrow i_c - i_d - i_b = 0$

Rearranging the equations gives,

$-i_a + 4i_b + i_c + 2i_d = 0$

$3i_a + 0i_b - i_c + 0i_d = 10$

$0i_a - i_b + i_c + 0i_d = 2$

$0i_a - i_b + i_c - i_d = 0$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A^{-1}Ax} = \boldsymbol{A^{-1}b} \Rightarrow \boldsymbol{x} = \boldsymbol{A^{-1}b}$

$$\begin{bmatrix} -1 & 4 & 1 & 2 \\ 3 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 2 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \end{bmatrix} = \begin{bmatrix} 3.857 \\ -0.4286 \\ 1.571 \\ 2 \end{bmatrix}$$

This implies that, $i_a = 3.8571A, i_b = -0.4286A, i_c = 1.5714A$ *and* $i_d = 2A$

Now it's clear that, $i_1 = i_b, i_2 = i_a - i_c$ *and* $i_3 = i_d$

So that, $i_1 = \boldsymbol{-0 \cdot 4286A}, i_2 = 2.286A$ *and* $i_3 = \boldsymbol{2A}$

Chapter 3- Problems

3.3

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_0 *gives,* $-10 + \frac{v_0}{10} + \frac{v_0}{20} + \frac{v_0}{30} + 2 + \frac{v_0}{60} = 0 \Rightarrow v_0 = 40V$

This implies that, $I_1 = \mathbf{4A}$, $I_2 = \mathbf{2A}$, $I_3 = \mathbf{1.3333A}$ *and* $I_4 = \mathbf{0.6667A}$

3.5

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive and lower middle node as reference node!

It's clear that, upper middle node voltage equals to v_0 *because they are connected to same pair of terminals! (if there is any confusion you can verify it using Kirchhoff's Voltage Law!)*

By KVL for upper middle node gives, $\frac{v_0 - 30}{2000} + \frac{v_0 - 20}{5000} + \frac{v_0}{4000} = 0 \Rightarrow v_0 = \mathbf{20V}$

3.7

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive and second lower middle node as reference node!

By KCL for second middle upper node gives, $-2 + \frac{V_x}{10} + \frac{V_x}{20} + 0.2V_x = 0 \Rightarrow V_x = \mathbf{5.714V}$

3.9

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive and lower middle node as reference node!

Let denote $v_{150\Omega}$ *as* v_1 *and* $v_{50\Omega}$ *as* v_s*!*

CONT...

By KCL for node v_s *gives,* $\frac{v_s-12}{250}+\frac{v_s}{50}+\frac{v_1}{150}=0$

It's clear that, $60I_b=v_s-v_1$ *and* $I_b=\frac{12-v_s}{250}$

Rearranging the equations gives,

$$v_1\left(\frac{1}{150}\right)+v_s\left(\frac{1}{250}+\frac{1}{50}\right)+0I_b=\frac{12}{250}$$

$$v_1-v_s+60I_b=0$$

$$0v_1+v_s\left(\frac{1}{250}\right)+I_b=\frac{12}{250}$$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow\boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow\boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix}\frac{1}{150} & \frac{6}{250} & 1\\ 1 & -1 & 60\\ 0 & \frac{1}{250} & 1\end{bmatrix}\begin{bmatrix}v_1\\ v_s\\ I_b\end{bmatrix}=\begin{bmatrix}\frac{12}{250}\\ 0\\ \frac{12}{250}\end{bmatrix}\Rightarrow\begin{bmatrix}v_1\\ v_s\\ I_b\end{bmatrix}=\begin{bmatrix}-0.2975\\ 2.0826\\ 0.03967\end{bmatrix}$$

This implies that, $I_b=0.03967A=\boldsymbol{39.67mA}$

3.11

By Ohm's Law, $v=i\cdot R\Rightarrow i=\frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i=\frac{v_{higher}-v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for upper middle node gives, $v_0-36+\frac{v_0}{2}+\frac{v_0+12}{4}=0\Rightarrow v_0=18.86V$

By the definition of power, $p=\frac{dw(q)}{dt}=\frac{dw(q)}{dq}\cdot\frac{dq(t)}{dt}=v\cdot i=\frac{v^2}{R}$

This implies that, $p_{1\Omega}=(36-v_0)^2=\boldsymbol{293.8W}$, $p_{2\Omega}=\frac{v_0^2}{2}=\boldsymbol{177.8W}$ *and* $v_{4\Omega}=\frac{(v_0+12)^2}{4}=\boldsymbol{238.1W}$

3.13

By Ohm's Law, $v=i\cdot R\Rightarrow i=\frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i=\frac{v_{higher}-v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_2 *gives,* $\frac{v_2+2-v_1}{2}+\frac{v_2}{4}-3=0$

Also it's clear that, $\frac{v_1}{8}=\frac{v_2+2-v_1}{2}$

CONT...

Rearranging the equations gives,

$$v_1\left(\frac{-1}{2}\right)+v_2\left(\frac{1}{2}+\frac{1}{4}\right)=2$$

$$v_1\left(\frac{1}{8}+\frac{1}{2}\right)+v_2\left(\frac{-1}{2}\right)=1$$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow \boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix}\frac{-1}{2} & \frac{3}{4}\\ \frac{5}{8} & \frac{-1}{2}\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}2\\ 1\end{bmatrix}\Rightarrow\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}8\\ 8\end{bmatrix}$$

This implies that, $v_1=v_2=\mathbf{8V}$

3.15

By Ohm's Law, $v=i\cdot R\Rightarrow i=\frac{v}{R}=v\cdot G$

As it known, current flows from a higher potential to lower!

This implies that, $i=\frac{v_{higher}-v_{lower}}{R}$

Let denote currents leaving a node positive and lower middle node as reference node!

Let denote first,second and middle upper nodes as, v_1, v_2 *and* v_3

It's obvious that $10V$ *independent voltage source connects two non-reference nodes!*

This implies that, $10V$ *source creates a supernode!*

By KCL for supernode gives, $6v_1+2+5v_2+3(v_2-v_3)=0$

By KCL for node v_3 *gives,* $-2+3(v_3-v_2)-4=0$

Also it's clear that, $v_1-v_2=10$

Rearranging the equations gives,

$$6v_1+8v_2-3v_3=-2$$

$$0v_1-3v_2+3v_3=6$$

$$v_1-v_2+0v_3=10$$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow \boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix}6 & 8 & -3\\ 0 & -3 & 3\\ 1 & -1 & 0\end{bmatrix}\begin{bmatrix}v_1\\ v_2\\ v_3\end{bmatrix}=\begin{bmatrix}-2\\ 6\\ 10\end{bmatrix}\Rightarrow\begin{bmatrix}v_1\\ v_2\\ v_3\end{bmatrix}=\begin{bmatrix}4.409\\ -5.909\\ -3.909\end{bmatrix}$$

This implies that, $v_1=4.909V$, $v_2=\ -5.091V$ *and* $v_3=-3.091V$

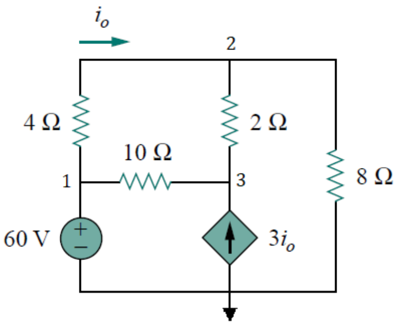
So that, $i_0=6v_1=\mathbf{29.45A}$

By the definition of power, $p=\frac{dw(q)}{dt}=\frac{dw(q)}{dq}\cdot\frac{dq(t)}{dt}=v\cdot i=\frac{v^2}{R}=v^2\cdot G$

This implies that, $p_{6S}=6v_1^2=\mathbf{144.6W}$, $p_{5S}=5v_2^2=\mathbf{129.6W}$ *and* $p_{3S}=3(v_3-v_2)^2=\mathbf{12W}$

3.17

Let redraw the circuit with some important voltages and a reference node!



By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for the node voltages 2 and 3 gives, $\frac{v_2 - v_1}{4} + \frac{v_2 - v_3}{2} + \frac{v_2}{8} = 0$ *and* $\frac{v_3 - v_1}{10} + \frac{v_3 - v_2}{2} - 3i_0 = 0$

It's clear that, $v_1 = 60V$ *and* $\frac{v_1 - v_2}{4} = i_0$

Rearranging the equations gives,

$$v_2\left(\frac{1}{4} + \frac{1}{2} + \frac{1}{8}\right) + v_3\left(-\frac{1}{2}\right) + 0i_0 = 15$$

$$v_2\left(-\frac{1}{2}\right) + v_3\left(\frac{1}{10} + \frac{1}{2}\right) - 3i_0 = 6$$

$$v_2\left(-\frac{1}{4}\right) + 0v_3 - i_0 = -15$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} \frac{7}{8} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{6}{10} & -3 \\ -\frac{1}{4} & 0 & -1 \end{bmatrix} \begin{bmatrix} v_2 \\ v_3 \\ i_0 \end{bmatrix} = \begin{bmatrix} 15 \\ 6 \\ -15 \end{bmatrix} \Rightarrow \begin{bmatrix} v_2 \\ v_3 \\ i_0 \end{bmatrix} = \begin{bmatrix} 53.08 \\ 62.88 \\ 1.73 \end{bmatrix}$$

This implies that, $i_0 = \mathbf{1.73A}$

3.19

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

CONT . . .

Let denote currents leaving a node positive!

By KCL for node voltages gives,

$$-5+3+\frac{v_1-v_3}{2}+\frac{v_1-v_2}{8}+\frac{v_1}{4}=0$$

$$\frac{v_2-v_1}{8}+\frac{v_2}{2}+\frac{v_2-v_3}{4}=0$$

$$-3+\frac{v_3-v_1}{2}+\frac{v_3-v_2}{4}+\frac{v_3-12}{8}=0$$

Rearranging the equations gives,

$$7v_1-v_2-4v_3=16$$

$$-v_1+7v_2-2v_3=0$$

$$-4v_1-2v_2+7v_3=36$$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow\boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow\boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix}7&-1&-4\\-1&7&-2\\-4&-2&7\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}16\\0\\36\end{bmatrix}\Rightarrow\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}10\\4.933\\12.267\end{bmatrix}$$

This implies that, $v_1=\mathbf{10V}$, $v_2=\mathbf{4.933V}$ *and* $v_3=\mathbf{12.267V}$

3.21

By Ohm's Law, $v=i\cdot R\Rightarrow i=\frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i=\frac{v_{higher}-v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives, $-0.003+\frac{v_1-v_2}{4000}+\frac{v_1+3v_0-v_2}{2000}=0$ *and* $\frac{v_2-v_1}{4000}+\frac{v_2-3v_0-v_1}{2000}+\frac{v_2}{1000}=0$

Also it's clear that, $v_2=v_0$*!*

Rearranging the equations gives,

$3v_1+3v_0=12$ *and* $-3v_1+v_0=0$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow\boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow\boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix}3&3\\-3&1\end{bmatrix}\begin{bmatrix}v_1\\v_0\end{bmatrix}=\begin{bmatrix}12\\0\end{bmatrix}\Rightarrow\begin{bmatrix}v_1\\v_0\end{bmatrix}=\begin{bmatrix}1\\3\end{bmatrix}$$

This implies that, $v_1=1V$ *and* $v_2=\mathbf{3V}$

3.23

By Ohm's Law, $v=i\cdot R\Rightarrow i=\frac{v}{R}$

As it known, current flows from a higher potential to lower!

CONT...

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive, lower left middle node as reference node and $v_{16\Omega}$ *as* v_1*!*

By KCL for node V_0 *gives,* $V_0 - 30 + \frac{V_0}{2} + \frac{V_0 - 2V_o - v_1}{4} = 0$

By KCL for node v_1 *gives,* $\frac{v_1 + 2V_0 - V_0}{4} + \frac{v_1}{16} - 3 = 0$

Rearranging the equations gives, $5V_0 - v_1 = 120$ *and* $4V_0 + 4v_1 = 48$

As it known, $\boldsymbol{Ax = b \Rightarrow A^{-1}Ax = A^{-1}b \Rightarrow x = A^{-1}b}$

$$\begin{bmatrix} \frac{5}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{5}{16} \end{bmatrix} \begin{bmatrix} V_0 \\ v_1 \end{bmatrix} = \begin{bmatrix} 30 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} V_0 \\ v_1 \end{bmatrix} = \begin{bmatrix} 22.34 \\ -8.275 \end{bmatrix}$$

This implies that, $V_0 = \boldsymbol{22.34V}$

3.25

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R} = v \cdot G$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for the node voltages gives,

$$-4 + \frac{v_1 - v_4}{20} + v_1 - v_2 = 0$$

$$v_2 - v_1 + \frac{v_2}{8} + \frac{v_2 - v_3}{10} = 0$$

$$\frac{v_3 - v_2}{10} + \frac{v_3 - v_4}{10} + \frac{v_3}{20} = 0$$

$$\frac{v_4 - v_1}{20} + \frac{v_4 - v_3}{10} + \frac{v_4}{30} = 0$$

Rearranging the equations gives,

$$v_1\left(\frac{1}{20} + 1\right) - v_2 + 0v_3 + v_4\left(-\frac{1}{20}\right) = 4$$

$$-v_1 + v_2\left(\frac{1}{10} + \frac{1}{8} + 1\right) + v_3\left(-\frac{1}{10}\right) + 0v_4 = 0$$

$$0v_1 + v_2\left(-\frac{1}{10}\right) + v_3\left(\frac{1}{10} + \frac{1}{10} + \frac{1}{20}\right) + v_4\left(-\frac{1}{10}\right) = 0$$

$$v_1\left(-\frac{1}{20}\right) + 0v_2 + v_3\left(-\frac{1}{10}\right) + v_4\left(\frac{1}{20} + \frac{1}{10} + \frac{1}{30}\right) = 0$$

As it known, $\boldsymbol{Ax = b \Rightarrow A^{-1}Ax = A^{-1}b \Rightarrow x = A^{-1}b}$

CONT...

$$\begin{bmatrix} \frac{21}{20} & -1 & 0 & 0 \\ -1 & \frac{49}{40} & -\frac{1}{10} & 0 \\ 0 & -\frac{1}{10} & \frac{1}{4} & -\frac{1}{10} \\ -\frac{1}{20} & 0 & -\frac{1}{10} & \frac{11}{60} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \end{bmatrix} = \begin{bmatrix} 25.52 \\ 22.05 \\ 14.84 \\ 15.06 \end{bmatrix}$$

This implies that, $v_1 = 25.52V, v_2 = 22.05V, v_3 = 14.84V$ *and* $v_4 = 15.06V$

3.27

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R} = v \cdot G$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive and lower middle node as reference node!

By KCL for node voltages gives,

$-2 + 4(v_1 - v_3) + 3i_0 + v_1 - v_2 + 2v_1 = 0$

$v_2 - v_1 + 4v_2 + v_2 - v_3 = 0$

$4(v_3 - v_1) - 3i_0 + v_3 - v_2 + 2v_3 - 4 = 0$

It's clear that, $i_0 = 4v_2$

Rearranging the equations with substitution gives,

$7v_1 + 11v_2 - 4v_3 = 2$

$-v_1 + 6v_2 - v_3 = 0$

$-4v_1 - 13v_2 + 7v_3 = 4$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 7 & 11 & -4 \\ -1 & 6 & -1 \\ -4 & -13 & 7 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0.625 \\ 0.375 \\ 1.625 \end{bmatrix}$$

This implies that, $v_1 = 0.625V = 625mV,\ v_2 = \mathbf{0.375V} = \mathbf{375mV}$ *and* $v_3 = \mathbf{1.625V}$

3.29

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R} = v \cdot G$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for the node voltages gives,

CONT...

$5 + v_1 - v_4 + 2v_1 + v_1 - v_2 = 0$

$v_2 - v_1 + 2v_2 + 4(v_2 - v_3) = 0$

$4(v_3 - v_2) + v_3 - v_4 - 6 = 0$

$v_4 - v_1 - 2 + 3v_4 + v_4 - v_3 = 0$

Rearranging the equations gives,

$4v_1 - v_2 + 0v_3 - v_4 = -5$

$-v_1 + 7v_2 - 4v_3 + 0v_4 = 0$

$0v_1 - 4v_2 + 5v_3 - v_4 = 6$

$-v_1 + 0v_2 - v_3 + 5v_4 = 2$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 4 & -1 & 0 & -1 \\ -1 & 7 & -4 & 0 \\ 0 & -4 & 5 & -1 \\ -1 & 0 & -1 & 5 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 6 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \end{bmatrix} = \begin{bmatrix} -0.7708 \\ 1.209 \\ 2.309 \\ 0.7076 \end{bmatrix}$$

This implies that,

$v_1 = -0.7708V = \boldsymbol{-770.8mV}, v_2 = \boldsymbol{1.209V}, v_3 = \boldsymbol{2.309V}$ *and* $v_4 = \boldsymbol{0.7076V} = \boldsymbol{707.6mV}$

3.31

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R} = v \cdot G$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

It's obvious that, $4I_0$ *independent voltage source connects two non-reference nodes!*

This implies that, source creates a supernode!

By KCL for supernode gives, $-1 + v_1 - v_3 + \frac{v_1}{4} + v_2 - 2v_0 = 0$

It's clear that, $v_1 - v_3 = v_0,\ v_2 - v_1 = 4I_0$ *and* $v_3 = 4I_0$

By KCL for node v_3 *gives,* $2v_0 + v_3 - v_1 + \frac{v_3}{4} + \frac{v_3 - 10}{2} = 0$

Rearraning the equations with substitution gives,

$v_1\left(1 + \frac{1}{4}\right) + v_2 - 2v_0 - 4I_0 = 1$

$v_1 + 0v_2 - v_0 - 4I_0 = 0$

$-v_1 + v_2 + 0v_0 - 4I_0 = 0$

$-v_1 + 0v_2 + 2v_0 + 7I_0 = 5$

CONT...

As it known, $\boldsymbol{Ax = b \Rightarrow A^{-1}Ax = A^{-1}b \Rightarrow x = A^{-1}b}$

$$\begin{bmatrix} 1.25 & 1 & -2 & -4 \\ 1 & 0 & -1 & -4 \\ -1 & 1 & 0 & -4 \\ -1 & 0 & 2 & 7 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_0 \\ I_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_2 \\ v_0 \\ I_0 \end{bmatrix} = \begin{bmatrix} 4.970 \\ 4.848 \\ 5.091 \\ -0.03 \end{bmatrix}$$

This implies that, $v_1 = 4.970V, v_2 = 4.848V, v_0 = 5.091V$ *and* $v_1 = -0.03A$

So that, $v_3 = 4I_0 = \boldsymbol{-0.12V = -120mV}$

3.39

Assume that direction of voltage drops signed positive!

By Ohm's Law, $v = i \cdot R$

By KVL for each mesh gives, $-10 + 4i_1 - 2I_x + 6(i_1 - i_2) = 0$ *and* $2i_2 + 12 + 6(i_2 - i_1) = 0$

Also it's clear that, $I_x = i_1 - i_2$ *(If there's any confusion, you can use Kirchhoff's Current Law!)*

Rearranging the equations gives,

$10i_1 - 6i_2 - 2I_x = 10$

$-6i_1 + 8i_2 + 0I_x = -12$

$-i_1 + i_2 + I_x = 0$

As it known, $\boldsymbol{Ax = b \Rightarrow A^{-1}Ax = A^{-1}b \Rightarrow x = A^{-1}b}$

$$\begin{bmatrix} 10 & -6 & -2 \\ -6 & 8 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ I_x \end{bmatrix} = \begin{bmatrix} 10 \\ -12 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ I_x \end{bmatrix} = \begin{bmatrix} 0.80 \\ -0.90 \\ 1.70 \end{bmatrix}$$

This implies that, $i_1 = \boldsymbol{0.80A = 800mA}$ *and* $i_2 = \boldsymbol{-0.90A = -900mA}$

3.41

Assume that direction of voltage drops signed positive!

By Ohm's Law, $v = i \cdot R$

By KVL for each mesh gives,

$10i_1 - 6 + 2(i_1 - i_2) = 0$

$2(i_2 - i_1) + i_2 - i_3 + 8 + 4i_2 = 0$

$6 + 5i_3 - 8 + i_3 - i_2 = 0$

Rearranging the equations gives,

$12i_1 - 2i_2 + 0i_3 = 6$

$-2i_1 + 7i_2 - i_3 = -8$

$0i_1 - i_2 + 6i_3 = 2$

As it known, $\boldsymbol{Ax = b \Rightarrow A^{-1}Ax = A^{-1}b \Rightarrow x = A^{-1}b}$

CONT...

$$\begin{bmatrix} 12 & -2 & 0 \\ -2 & 7 & -1 \\ 0 & -1 & 6 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0.3291 \\ -1.026 \\ 0.1624 \end{bmatrix}$$

This implies that, $i_1 = 0.3291A = 329.1mA,\ i_2 = -1.026A$ *and* $i_3 = 0.1624A = 162.4mA$

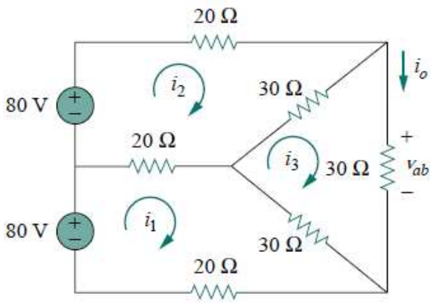
As it known, $i = i_3 - i_2 = \mathbf{1.188A}$

3.43

Assume that direction of voltage drops signed positive!

By Ohm's Law, $v = i \cdot R$

Let redraw the circuit with mesh currents!



By KVL for each mesh gives,

$-80 + 20(i_1 - i_2) + 30(i_1 - i_3) + 20i_1 = 0$

$-80 + 20i_2 + 30(i_2 - i_3) + 20(i_2 - i_1) = 0$

$30i_3 + 30(i_3 - i_1) + 30(i_3 - i_2) = 0$

Rearranging the equations gives,

$70i_1 - 20i_2 - 30i_3 = 80$

$-20i_1 + 70i_2 - 30i_3 = 80$

$-30i_1 - 30i_2 + 90i_3 = 0$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 70 & -20 & -30 \\ -20 & 70 & -30 \\ -30 & -30 & 90 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 80 \\ 80 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 2.6667 \\ 2.6667 \\ 1.7778 \end{bmatrix}$$

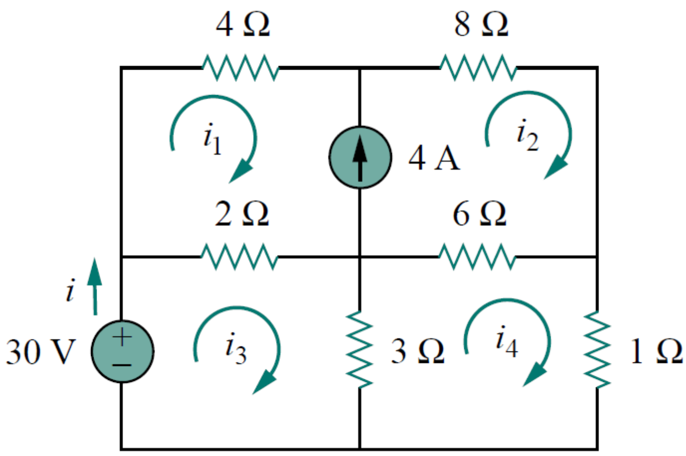
This implies that, $i_3 = \mathbf{1.7778A}$

It's clear that, $i_0 = i_3 = \mathbf{1.778A}$

Now, $v_{ab} = 30i_0 = \mathbf{53.33V}$

Let redraw the circuit with mesh currents!

CONT...



It's clear that, independent current source is in common with two different meshes, this implies that, there exist a supermesh!

Assume that direction of voltage drops signed positive!

By Ohm's Law, $v = i \cdot R$

By KVL for supermesh gives, $4i_1 + 8i_2 + 6(i_2 - i_4) + 2(i_1 - i_3) = 0$

By KVL for bottom meshes gives, $-30 + 2(i_3 - i_1) + 3(i_3 - i_4) = 0$

$6(i_4 - i_2) + i_4 + 3(i_4 - i_3) = 0$

It's clear that, $i_2 - i_1 = 4$

Rearranging the equations gives,

$6i_1 + 14i_2 - 2i_3 - 6i_4 = 0$

$-2i_1 + 0i_2 + 5i_3 - 3i_4 = 30$

$0i_1 - 6i_2 - 3i_3 + 10i_4 = 0$

$-i_1 + i_2 + 0i_3 + 0i_4 = 4$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

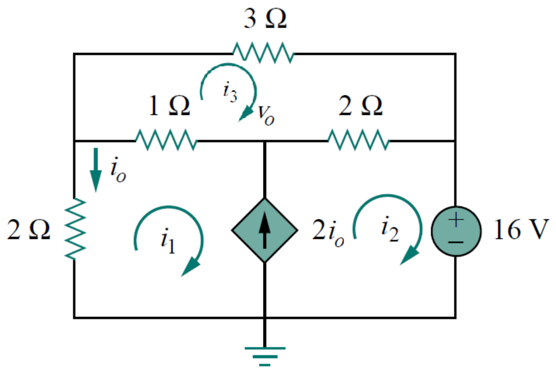
$$\begin{bmatrix} 6 & 14 & -2 & -6 \\ -2 & 0 & 5 & -3 \\ 0 & -6 & -3 & 10 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 30 \\ 0 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0.5530 \\ 3.447 \\ 8.561 \\ 4.636 \end{bmatrix}$$

This implies that, $i_3 = \mathbf{8.561}\boldsymbol{A}$

It's clear that, $i = i_3 = \mathbf{8.561}\boldsymbol{A}$

3.49

Let redraw the circuit with mesh currents!



It's clear that, dependent current source is in common with two different meshes, this implies that, there exist a supermesh!

Assume that direction of voltage drops signed positive!

By Ohm's Law, $v = i \cdot R$

By KVL for supermesh gives, $i_1 - i_3 + 2(i_2 - i_3) + 16 + 2i_1 = 0$

By KVL for upper mesh gives, $3i_3 + 2(i_3 - i_2) + i_3 - i_1 = 0$

It's clear that, $i_2 - i_1 = 2i_0$ *and* $i_0 = -i_1$

Rearranging the equations with substitution gives,

$3i_1 + 2i_2 - 3i_3 = -16$

$-i_1 - 2i_2 + 6i_3 = 0$

$i_1 + i_2 + 0i_3 = 0$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 3 & 2 & -3 \\ -1 & -2 & 6 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -16 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -10.67 \\ 10.67 \\ 1.78 \end{bmatrix}$$

This implies that, $i_1 = -10.67A$, $i_2 = 10.67A$ *and* $i_3 = 1.78A$

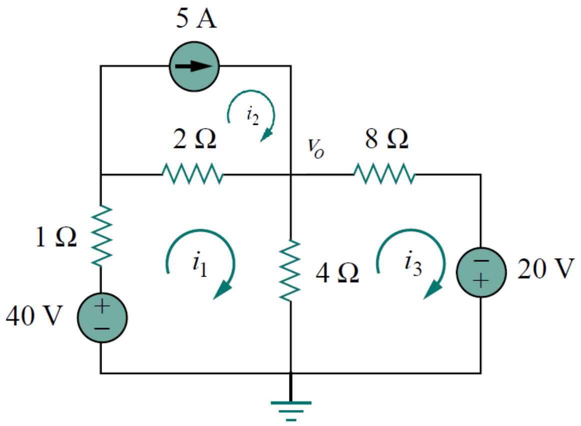
So that, $i_0 = -i_1 = 10.67A$

Let denote currents leaving a node positive!

This implies that, $\frac{v_o - 16}{2} = i_2 - i_3 \Rightarrow v_o = \mathbf{33.78V}$

3.51

Let redraw the circuit with mesh currents!



Assume that direction of voltage drops signed positive!

By Ohm's Law, $v = i \cdot R$

By KVL for bottom meshes gives,

$-40 + i_1 + 2(i_1 - i_2) + 4(i_1 - i_3) = 0$

$8i_3 - 20 + 4(i_3 - i_1) = 0$

It's clear that, $i_2 = \mathbf{5}\boldsymbol{A}$

Rearranging the equations with substitution gives,

$7i_1 - 4i_3 = 50$

$-4i_1 + 12i_3 = 20$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$\begin{bmatrix} 7 & -4 \\ -4 & 12 \end{bmatrix}\begin{bmatrix} i_1 \\ i_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 20 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

This implies that, $i_1 = 10A$ *and* $i_3 = 5A$

It's clear that, $\frac{v_0}{4} = i_1 - i_3 \Rightarrow v_0 = 4(i_1 - i_3) = \mathbf{20}\boldsymbol{V}$

3.53

Assume that direction of voltage drops signed positive!

By Ohm's Law, $v = i \cdot R$

By KVL for each mesh gives,

$-12 + 1000(I_1 - I_3) + 3000(I_1 - I_2) = \mathbf{0}$

$4000(I_2 - I_4) + 3000(I_2 - I_1) = \mathbf{0}$

CONT...

$6000(I_3 - I_5) + 8000(I_3 - I_4) + 1000(I_3 - I_1) = \mathbf{0}$

$2000I_5 + 8000(I_5 - I_4) + 6000(I_5 - I_3) = \mathbf{0}$

It's clear that, $I_4 = 3mA$

Rearranging the equations with substitution gives,

$4I_1 - 3I_2 - 1I_3 + 0I_5 = 12$

$-3I_1 + 7I_2 + 0I_3 + 0I_5 = -12$

$I_1 + 0I_2 - 15I_3 + 6I_5 = 24$

$0I_1 + 0I_2 - 6I_3 + 16I_5 = 24$

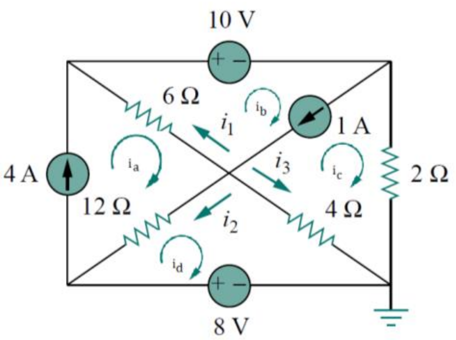
As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 4 & -3 & -1 & 0 \\ -3 & 7 & 0 & 0 \\ 1 & 0 & -15 & 6 \\ 0 & 0 & -6 & 16 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_5 \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \\ 24 \\ 24 \end{bmatrix} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_5 \end{bmatrix} = \begin{bmatrix} 1.6196 \\ -1.0202 \\ -2.4612 \\ -2.4230 \end{bmatrix}$$

This implies that, $i_1 = 1.6196mA$, $I_2 = \mathbf{1.6196}\boldsymbol{mA}$, $I_3 = \mathbf{1.6196}\boldsymbol{mA}$ *and* $I_5 = \mathbf{1.6196}\boldsymbol{mA}$

3.55

Let redraw the circuit with mesh currents!



By Ohm's Law, $v = i \cdot R$

Assume that direction of voltage drops signed positive!

It's clear that, independent current source is in common with two different meshes, this implies that, there exist a supermesh!

By KVL for supermesh gives, $10 + 2i_c + 4(i_c - i_d) + 6(i_b - i_a) = 0$

By KVL for bottom mesh gives, $4(i_d - i_c) - 8 + 12(i_d - i_a) = 0$

It's clear that, $i_a = 4A$ *and* $i_b - i_c = 1$

Rearranging the equations with substitution gives,

$6i_b + 6i_c - 4i_d = 14$

$0i_b - 4i_c + 16i_d = 56$

$i_b - i_c + 0i_d = 1$

CONT . . .

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 6 & 6 & -4 \\ 0 & -4 & 16 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} i_b \\ i_c \\ i_d \end{bmatrix} = \begin{bmatrix} 14 \\ 56 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} i_b \\ i_c \\ i_d \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$$

This implies that, $i_b = 3A,\ i_c = 2A$ *and* $i_d = 4A$

It's clear that, $i_1 = i_b - i_a = \boldsymbol{-1A}$ *,* $i_2 = i_d - i_a = \boldsymbol{0A}$ *and* $i_3 = i_d - i_c = \boldsymbol{2A}$

3.55

It's clear that R *and* $3k\Omega$ *resistors are connected in parallel!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

This implies that, R_{eq} *for* R *and* $3k\Omega$ *resistors is* $R_{eq} = \left(\frac{1}{3} + \frac{1}{R}\right)^{-1} = \frac{3 \cdot R}{3+R}$

It's clear that, R_{eq} *for* R *and* $3k\Omega$ *is connected in series with* $4k\Omega$ *resistor!*

This implies, $R_{eq} = \frac{3 \cdot R}{3+R} + 4$

It's clear that $100 = v_{R_{eq}}$ *because they are both connected to the same pair of terminals!(if there is any confusion you can verify it using Kirchhoff's Voltage Law!)*

By Ohm's Law, $v = i \cdot R \Rightarrow R = \frac{v}{i}$

This implies that, $R_{eq} = \frac{100}{18 \times 10^{-3}} = \boldsymbol{5.556k\Omega}$

As it known, $5.556 - 4 = \frac{3 \cdot R}{3+R} \Rightarrow R = \boldsymbol{3.23k\Omega}$

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

This implies that, $V_1 = \frac{1.56}{5556} 100V = \boldsymbol{28V}$ *and* $V_2 = \frac{4}{5.556} 100V = \boldsymbol{72V}$

3.61

By Ohm's Law, $v = i \cdot R$

Assume that direction of voltage drops signed positive!

Let denote the mesh currents for the consecutive closed paths as i_1, i_2 *and* i_3*!*

It's clear that, $i_1 = i_s,\ i_3 = i_0$ *and* $v_0 = 30(i_1 - i_2)$

By KVL for other meshes gives, $20i_2 - 5v_0 + 30(i_2 - i_1) = 0$ *and* $10i_3 + 40i_3 + 5v_0 = 0$

Rearranging the equations with substitution gives,

CONT...

$-180i_s + 200i_2 = 0$

$-150i_2 + 50i_0 + 150i_s = 0$

Now multiply both equations with the numbers needed!

$-540i_s + 600i_2 = 0$

$-600i_2 + 200i_0 + 600i_s = 0$

Now in order to get a new equation add them up!

$60i_s = -200i_0$

This implies that, current gain becomes $\frac{i_0}{i_s} = \mathbf{-0.3}$

3.63

By Ohm's Law, $v = i \cdot R$

Assume that direction of voltage drops signed positive!

Let denote the mesh currents for the first and last closed paths as i_1 *and* i_2*!*

It's clear that, both the independent and dependent current source are in common with two different meshes, this implies that, there exist a supermesh!(Note that super mesh can be applied as single larger supermesh!)

By KVL for larger supermesh gives, $10i_1 + 5i_2 + 4i_x - 50 = 0$

Let denote currents leaving a node positive!

By KCL for upper second middle node gives, $-i_x - 3 - \frac{v_x}{4} + i_2 = 0$

It's clear that, $i_1 = i_x$ *and* $v_x = 2(i_1 - i_2)$

Rearranging the equations with substitution gives,

$14i_x + 5i_2 + 0v_x = 50$

$-i_x + i_2 - \frac{v_x}{4} = 3$

$-2i_x + 2i_2 + v_x = 0$

As it known, $\boldsymbol{Ax = b \Rightarrow A^{-1}Ax = A^{-1}b \Rightarrow x = A^{-1}b}$

$$\begin{bmatrix} 14 & 5 & 0 \\ -1 & 1 & -\frac{1}{4} \\ -2 & 2 & 1 \end{bmatrix} \begin{bmatrix} i_x \\ i_2 \\ v_x \end{bmatrix} = \begin{bmatrix} 50 \\ 3 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_x \\ i_2 \\ v_x \end{bmatrix} = \begin{bmatrix} 2.105 \\ 4.105 \\ -4 \end{bmatrix}$$

This implies that, $i_x = \mathbf{2.105A}$ *and* $v_x = \mathbf{-4V}$

3.65

By Ohm's Law, $v = i \cdot R$

Assume that direction of voltage drops signed positive!

CONT...

By KVL for each mesh gives,

$-12 + 5i_1 + i_1 - i_4 + 6(i_1 - i_2) = 0$

$i_2 - i_4 + i_2 - i_5 + 8(i_2 - i_3) + 6(i_2 - i_1) = 0$

$i_3 - i_5 + 6i_3 - 9 + 8(i_3 - i_2) = 0$

$-6 + 3i_4 + 2(i_4 - i_5) + i_4 - i_2 + i_4 - i_1 = 0$

$4i_5 - 10 + i_5 - i_3 + i_5 - i_2 + 2(i_5 - i_4) = 0$

Rearranging the equations gives,

$12i_1 - 6i_2 + 0i_3 - i_4 + 0i_5 = 12$

$-6i_1 + 16i_2 - 8i_3 - i_4 - i_5 = 0$

$0i_1 - 8i_2 + 15i_3 + 0i_4 - i_5 = 9$

$-i_1 - i_2 + 0i_3 + 7i_4 - 2i_5 = 6$

$0i_1 - i_2 - i_3 - 2i_4 + 8i_5 = 10$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 12 & -6 & 0 & -1 & 0 \\ -6 & 16 & -8 & -1 & -1 \\ 0 & -8 & 15 & 0 & -1 \\ -1 & -1 & 0 & 7 & -2 \\ 0 & -1 & -1 & -2 & 8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 9 \\ 6 \\ 10 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 2.17 \\ 1.99 \\ 1.81 \\ 2.09 \\ 2.25 \end{bmatrix}$$

This implies that, $i_1 = \mathbf{2.17A}$, $i_2 = \mathbf{1.99A}$, $i_3 = \mathbf{1.81A}$, $i_4 = 2.09A$ *and* $i_5 = \mathbf{2.25A}$

3.67

Let denote $v_{10\Omega}$ *as* v_1, $v_{5\Omega}$ *as* v_2 *and* v_{4A} *as* v_3

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R} = v \cdot G$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives,

$-3V_0 + 2 + \dfrac{v_1 - v_2}{4} + \dfrac{v_1}{10} = 0$

$\frac{v_2 - v_1}{4} + \frac{v_2}{5} + \frac{v_2 - v_3}{2} = 0$

$\dfrac{v_3 - v_2}{2} - 2 - 4 = 0$

It's clear that, $v_2 - v_3 = V_0$

CONT...

Rearranging the equations gives,

$$v_1\left(\frac{1}{4}+\frac{1}{10}\right)+v_2\left(-\frac{1}{4}\right)+0v_3-3V_0=-2$$

$$v_1\left(-\frac{1}{4}\right)+v_2\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{2}\right)+v_3\left(-\frac{1}{2}\right)-3V_0=0$$

$$0v_1+v_2\left(-\frac{1}{2}\right)+v_3\left(\frac{1}{2}\right)+0V_0=6$$

$$0v_1+v_2-v_3-V_0=0$$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow \boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} \frac{7}{20} & -\frac{1}{4} & 0 & -3 \\ -\frac{1}{4} & \frac{19}{20} & -\frac{1}{2} & -3 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ V_0 \end{bmatrix}=\begin{bmatrix} -2 \\ 0 \\ 6 \\ 0 \end{bmatrix}\Rightarrow\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ V_0 \end{bmatrix}=\begin{bmatrix} -258.95 \\ -210.53 \\ -198.53 \\ -12 \end{bmatrix}$$

This implies that, $V_0=\mathbf{-12V}$

3.69

By Ohm's Law, $v=i\cdot R\Rightarrow i=\frac{v}{R}=v\cdot G$

As it known, current flows from a higher potential to lower!

This implies that, $i=\frac{v_{higher}-v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives,

$$-0.02+\frac{v_1-v_3}{1000}+\frac{v_1-v_2}{4000}+\frac{v_1}{2000}=0$$

$$\frac{v_2-v_1}{4000}-0.005+\frac{v_2-v_3}{4000}+\frac{v_2}{2000}=0$$

$$\frac{v_3-v_2}{4000}+0.005+\frac{v_3-v_1}{1000}-0.010=0$$

Rearranging the equations gives,

$$v_1\left(1+\frac{1}{4}+\frac{1}{2}\right)+v_2\left(-\frac{1}{4}\right)+v_3(-1)=\mathbf{20}$$

$$v_1\left(-\frac{1}{4}\right)+v_2\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{2}\right)+v_3\left(-\frac{1}{4}\right)=\mathbf{5}$$

$$-v_1+v_2\left(-\frac{1}{4}\right)+v_3\left(\frac{1}{4}+1\right)=\mathbf{5}$$

CONT . . .

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 1.75 & -0.25 & -1 \\ -0.25 & 1 & -0.25 \\ -1 & -0.25 & 1.25 \end{bmatrix} \begin{bmatrix} i_x \\ i_2 \\ v_x \end{bmatrix} = \begin{bmatrix} 20 \\ 5 \\ 5 \end{bmatrix}$$

So that, currents can be found by solving the matrix above!

3.71

By Ohm's Law, $v = i \cdot R$

Assume that direction of voltage drops signed positive!

By KVL for each mesh gives,

$-10 + 5(i_1 - i_3) + 4(i_1 - i_2) = 0$

$i_2 - i_3 + 2i_2 + 5 + 4(i_2 - i_1) = 0$

$3i_3 + i_3 - i_2 + 5(i_3 - i_1) = 0$

Rearranging the equations gives,

$9i_1 - 4i_2 - 5i_3 = 10$

$-4i_1 + 7i_2 - i_3 = -5$

$-5i_1 - i_2 + 9i_3 = 0$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 9 & -4 & -5 \\ -4 & 7 & -1 \\ -5 & -1 & 9 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 2.085 \\ 0.6533 \\ 1.231 \end{bmatrix}$$

This implies that, $i_1 = \mathbf{2.085A}$, $i_2 = 0.6533A = \mathbf{653.3mA}$ *and* $i_3 = \mathbf{1.231A}$

3.73

By Ohm's Law, $v = i \cdot R$

Assume that direction of voltage drops signed positive!

By KVL for each mesh gives,

$-6 + 2i_1 + 3(i_1 - i_2) + 4(i_1 - i_3) = 0$

$5i_2 - 4 + 3(i_2 - i_1) = 0$

$4(i_3 - i_1) + i_3 - i_4 - 2 + i_3 = 0$

$i_4 + 3 + i_4 - i_3 = 0$

Rearranging the equations gives,

$9i_1 - 3i_2 - 4i_3 + 0i_4 = 6$

$-3i_1 + 8i_2 + 0i_3 + 0i_4 = 4$

CONT...

$-4i_1 + 0i_2 + 6i_3 - i_4 = 2$

$0i_1 + 0i_2 - i_3 + 2i_4 = -3$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 9 & -3 & -4 & 0 \\ -3 & 8 & 0 & 0 \\ -4 & 0 & 6 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 2 \\ -3 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

So that, currents can be found by solving the matrix above!

3.77

Note that PSpice must be used but in order to make everything clear, regular process will be applied!

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R} = v \cdot G$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives, $-5 + 2i_x + \frac{V_1 - V_2}{5} + \frac{V_1}{2} = 0$ *and* $\frac{V_2 - V_1}{5} - 2i_x + 2 + V_2 = 0$

It's clear that, $i_x = \frac{V_1}{2}$

Rearranging the equations gives,

$v_1\left(\frac{1}{5} + \frac{1}{2}\right) + v_2\left(-\frac{1}{5}\right) + 2i_x = 5$

$v_1\left(-\frac{1}{5}\right) + v_2\left(\frac{1}{5} + 1\right) - 2i_x = -2$

$v_1\left(-\frac{1}{2}\right) + 0v_2 + i_x = 0$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 0.7 & -0.2 & 2 \\ -0.2 & 1.2 & -2 \\ -0.5 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ i_x \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} V_1 \\ V_2 \\ i_x \end{bmatrix} = \begin{bmatrix} 3.111 \\ 1.444 \\ 1.556 \end{bmatrix}$$

This implies that, $V_1 = 3.111V$ *and* $V_2 = 1.444V$

END!

Chapter 4-Practice Problems

4.1

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R} =$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

Let denote $v_{4\Omega}$ *as* v_1*!*

By KCL for upper middle node gives, $-i_s + \frac{v_1}{4} + \frac{v_1 - v_0}{12} = 0$

It's clear that, $\frac{v_1 - v_0}{12} = \frac{v_0}{8}$

When $i_s = 15A$*, rearranging the equations with substitution gives,*

$$v_1\left(\frac{1}{4} + \frac{1}{12}\right) + v_0\left(-\frac{1}{12}\right) = 15$$

$$v_1\left(\frac{1}{12}\right) + v_0\left(-\frac{1}{12} - \frac{1}{8}\right) = 0$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} \frac{1}{3} & -\frac{1}{12} \\ \frac{1}{12} & -\frac{5}{24} \end{bmatrix} \begin{bmatrix} v_1 \\ v_0 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_0 \end{bmatrix} = \begin{bmatrix} 50 \\ 20 \end{bmatrix}$$

This implies that, $v_0 = \boldsymbol{20V}$

So that when $i_s = \boldsymbol{30A}$

$$\begin{bmatrix} \frac{1}{3} & -\frac{1}{12} \\ \frac{1}{12} & -\frac{5}{24} \end{bmatrix} \begin{bmatrix} v_1 \\ v_0 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_0 \end{bmatrix} = \begin{bmatrix} 100 \\ 40 \end{bmatrix}$$

This implies that, $v_0 = \boldsymbol{40V}$

4.2

By Ohm's Law, $v = i \cdot R$

Assume that direction of voltage drops signed positive!

Let denote the mesh currents for the first and second closed paths as i_1 *and* i_2*!*

Assuming V_0 *as* $1V$ *gives,* $1 = 8i_2 \Rightarrow i_2 = 0.125A = 125mA$

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections!*

CONT...

This implies that, R_{eq} *for* 12Ω *and* 8Ω *is* $R_{eq} = 12\Omega + 8\Omega = 20\Omega$

Now, $v_{5\Omega} = 20i_2 = \mathbf{2.5V}$

Assuming V_0 *as* $1V$ *gives independent voltage source voltage as* $2.5V$*, so that actual value of source* $30V$ *will give* $V_0 = \mathbf{12V}$

4.3

Let $v_0 = v_1 + v_2$ *where* v_1 *and* v_2 *are the contributions due to the* $4A$ *current and* $10V$ *voltage sources,respectively!*

To obtain v_1 *set voltage source as zero!*

Let denote v_{4A} *as* v_x*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R} =$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for upper middle node gives, $\frac{v_x - v_1}{3} - 4 + \frac{v_x}{5} = 0$

It's clear that, $\frac{v_1}{2} = \frac{v_x - v_1}{3}$

Rearranging the equations gives,

$$v_1\left(-\frac{1}{3}\right) + v_x\left(\frac{1}{3} + \frac{1}{5}\right) = 4$$

$$v_1\left(\frac{1}{2} + \frac{1}{3}\right) + v_x\left(-\frac{1}{3}\right) = 0$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} -\frac{1}{3} & \frac{8}{15} \\ \frac{5}{6} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_x \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_x \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$$

This implies that, $v_1 = \mathbf{4V}$

To obtain v_2 *set current source as zero!*

Let denote the current that flows in the circuit as i_s*!*

Assume that direction of voltage drops signed positive!

By KVL for closed path gives, $2i_s + 3i_s + 5i_s + 10 = 0 \Rightarrow i_s = \mathbf{-1A}$

This implies that, $v_2 = -2i_s = \mathbf{2}V$

So that, $v_0 = v_1 + v_2 = 4V + 2V = \mathbf{6V}$

4.4

Let $v_x = v_1 + v_2$ *where* v_1 *and* v_2 *are the contributions due to the* $20V$ *voltage and* $4A$ *current sources, respectively!*

To obtain v_1 *set current source as zero!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R} =$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_1 *gives,* $\frac{v_1 - 20}{20} + \frac{v_1}{4} - 0.1v_1 = 0$

This implies that, $v_1 = \mathbf{5V}$

To obtain v_2 *set voltage source as zero!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_2 *gives,* $\frac{v_2}{20} - 4 + \frac{v_2}{4} - 0.1v_2 = 0$

This implies that, $v_2 = \mathbf{20V}$

So that, $v_x = v_1 + v_2 = 5V + 20V = \mathbf{25V}$

4.5

Let $I = i_1 + i_2 + i_3$ *where* i_1*,* i_2 *and* i_3 *are the contributions due to the* $16V$ *voltage,* $4A$ *and* $12V$ *current sources, respectively!*

To obtain i_1 *set* $4A$ *and* $12V$ *sources as zero!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

Assume that direction of voltage drops signed positive!

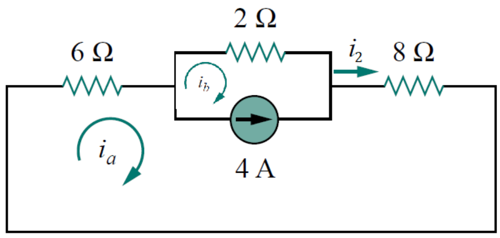
By KVL for closed path gives, $-16 + 6i_1 + 2i_1 + 8i_1 = 0$

This implies that, $i_1 = 1A$

To obtain i_2 *set* $16V$ *and* $12V$ *sources as zero!*

Let redraw the circuit with mesh currents!

CONT...



Assume that direction of voltage drops signed positive!

It's clear that, independent current source is in common with two different meshes, this implies that, there exist a supermesh!

By KVL for supermesh gives, $6i_a + 2i_b + 8i_a = 0$

It's clear that, $i_a - i_b = \mathbf{4}$

As it known, $\boldsymbol{Ax = b \Rightarrow A^{-1}Ax = A^{-1}b \Rightarrow x = A^{-1}b}$

$$\begin{bmatrix} 14 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 0.5 \\ -3.5 \end{bmatrix}$$

This implies that, $i_a = i_2 = 0.5A$

To obtain i_3 *set* $16V$ *and* $4A$ *sources as zero!*

By KVL for closed path gives, $6i_3 + 2i_3 + 8i_3 + 12 = 0$

This implies that, $i_3 = \mathbf{-0.75A}$

So that, $I = i_1 + i_2 + i_3 = 1A + 0.5A - 0.75A = \mathbf{0.75A}$

4.6

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

It's clear that, 1Ω *and* 4Ω *are connected in series so* R_{eq} *for them is* $1\Omega + 4\Omega = 5\Omega$

Now, 6Ω *and* 3Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{6} + \frac{1}{3} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 2\Omega$

It's obvious that, $5A$ *source and* 2Ω *resistor is connected in parallel so that, they can be replaced with* $10V$ *source and* 2Ω *resistor connected in series!*

Now, it's clear that, $10V$ *and* $5V$ *sources connected in series with same polarity!*

This implies that, we can assume that situation as single $15V$ *source!*

It's obvious that, $15V$ *source and* 2Ω *resistor are connected in series so that, they can be replaced with* $7.5A$ *source and* 2Ω *resistor connected in parallel!*

CONT...

Let denote source current as i_s*!*

Now, in order to create single source current, just add two independent current sources!

This implies that, $i_S = 7.5 + 3 = 10.5A$

By Current Division equation, $i_x = \frac{R_{eq}}{R_x} i_{source}$

It's clear that, $2\Omega, 7\Omega$ *and* 5Ω *resistor are connected in parallel so that,*

R_{eq} *for them is* $\frac{1}{2} + \frac{1}{7} + \frac{1}{5} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = \mathbf{1.186\Omega}$

This implies, $i_0 = \frac{1.186}{7} 10.5 = \mathbf{1.78A}$

4.7

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

It's obvious that, $2i_x$ *source and* 5Ω *resistor is connected in series so that, they can be replaced with* $0.4A$ *source and* 5Ω *resistor connected in parallel!(Note that direction must be downward!)*

Let denote source current as i_s*!*

Now, in order to create single source current, just add two independent current sources!

This implies that, $i_S = 0.024 - 0.4i_x$

By Current Division equation, $i_x = \frac{R_{eq}}{R_x} i_{source}$

It's clear that, 10Ω *and* 5Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{10} + \frac{1}{5} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = \mathbf{3.333\Omega}$

This implies, $i_x = \frac{3.333}{10}(0.024 - 0.4ix) \Rightarrow i_x = 7.056 \times 10^{-3}A = \mathbf{7.056mA}$

4.8

Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{ab} = v_{4\Omega}$ *directly and voltage drop across the terminals of* 1Ω *resistor is zero!*

Let denote v_{3A} *as* v_1*!*

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

CONT...

By KCL for node voltages gives,

$$\frac{v_1 - 18}{6} - 3 + \frac{v_1 - v_{Th}}{6} = 0$$

$$\frac{v_{Th} - v_1}{6} + \frac{v_{Th}}{4} = 0$$

Rearranging the equations gives,

$$v_1\left(\frac{1}{6} + \frac{1}{6}\right) + v_{Th}\left(-\frac{1}{6}\right) = 6$$

$$v_1\left(-\frac{1}{6}\right) + v_{Th}\left(\frac{1}{6} + \frac{1}{4}\right) = 0$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} \frac{1}{3} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{5}{12} \end{bmatrix} \begin{bmatrix} v_1 \\ v_{Th} \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_{Th} \end{bmatrix} = \begin{bmatrix} 22.5 \\ 9 \end{bmatrix}$$

This implies that, $v_{Th} = 9V$

Now, let create a short-circuit for the terminals a and b!

Let denote short-circuit current as i_{sc}*!*

This implies that, voltage drop across the 4Ω *resistor becomes zero!*

Let denote v_{3A} *as* v_1*!*

By KCL for node v_1 *gives,* $\frac{v_1 - 18}{6} - 3 + \frac{v_1}{6} = 0 \Rightarrow v_1 = \mathbf{18V}$

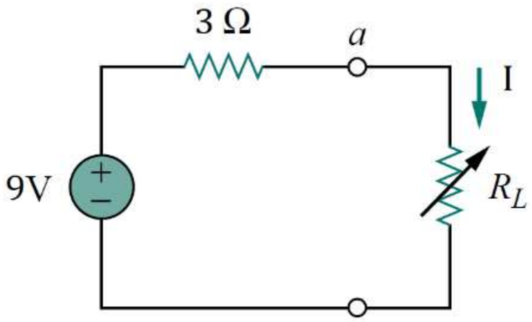
Now it's clear that, voltage across the first 6Ω *resistor is zero!*

Let denote currents leaving a node positive!

By KCL for node a gives, $-3 + i_s = 0 \Rightarrow i_s = \mathbf{3A}$

As it known, $R_{Th} = \frac{v_{Th}}{i_s} = \frac{9}{3} = \mathbf{3\Omega}$

So that the Thevenin equivalent circuit is the given below!



Here, R_L *corresponds to* 1Ω *resistor!*

By KVL for closed path gives, $-9 + 3I + I = 0 \Rightarrow I = \mathbf{2.25A}$

4.9

Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{ab} = v_{4\Omega}$ *directly!*

Let denote $v_{1.5I_x}$ *as* v_1*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives,

$$\frac{v_1 - 6}{5} - 1.5I_x + \frac{v_1 - v_{Th}}{3} = 0$$

$$\frac{v_{Th} - v_1}{3} + \frac{v_{Th}}{4} = 0$$

Also, it's clear that, $I_x = \frac{v_1 - v_{Th}}{3}$

Rearranging the equations gives,

$$v_1\left(\frac{1}{5} + \frac{1}{3}\right) + v_{Th}\left(-\frac{1}{3}\right) - 1.5I_x = \frac{6}{5}$$

$$v_1\left(-\frac{1}{3}\right) + v_{Th}\left(\frac{1}{3} + \frac{1}{4}\right) + 0I_x = 0$$

$$v_1\left(-\frac{1}{3}\right) + v_{Th}\left(\frac{1}{3}\right) + I_x = 0$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} \frac{8}{15} & -\frac{1}{3} & -1.5 \\ -\frac{1}{3} & \frac{7}{12} & 0 \\ -\frac{1}{3} & \frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_{Th} \\ I_x \end{bmatrix} = \begin{bmatrix} \frac{6}{5} \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_{Th} \\ I_x \end{bmatrix} = \begin{bmatrix} 9.33 \\ 5.33 \\ 1.33 \end{bmatrix}$$

This implies that, $v_{Th} = \boldsymbol{5.33V}$

Let create a short-circuit for the terminals a and b in order to find short-circuit current i_{sc}*!*

This implies that, voltage drop across the 4Ω *resistor becomes zero!*

Let denote $v_{1.5I_x}$ *as* v_1*!*

By KCL for node voltages gives,

$$\frac{v_1 - 6}{5} - 1.5I_x + \frac{v_1}{3} = 0$$

CONT...

Also, it's clear that, $I_x = \frac{\boldsymbol{v_1}}{\boldsymbol{3}}$*!*

It's obvious that, $i_{sc} = I_x$

Rearranging the equations with substitution gives,

$$v_1\left(\frac{1}{5}+\frac{1}{3}\right) - 1.5i_s = \frac{6}{5}$$

$$v_1\left(-\frac{1}{3}\right) + i_s = \mathbf{0}$$

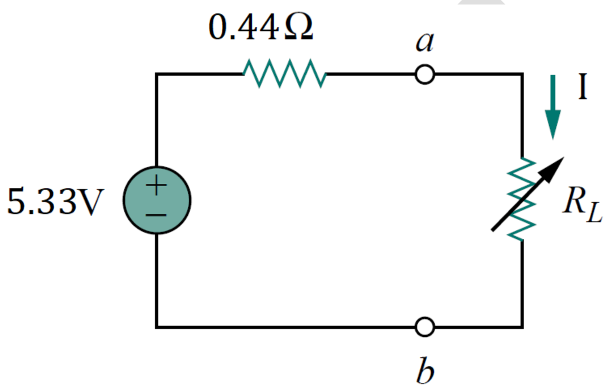
As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A^{-1}Ax} = \boldsymbol{A^{-1}b} \Rightarrow \boldsymbol{x} = \boldsymbol{A^{-1}b}$

$$\begin{bmatrix} \frac{8}{15} & -1.5 \\ -\frac{1}{3} & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ i_s \end{bmatrix} = \begin{bmatrix} \frac{6}{5} \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ i_s \end{bmatrix} = \begin{bmatrix} 36 \\ 12 \end{bmatrix}$$

This implies that, $i_s = 12A$

As it known, $R_{Th} = \frac{v_{Th}}{i_s} = \frac{5.33}{12} = \boldsymbol{0.44\Omega}$

So that the Thevenin equivalent circuit is the given below!



4.10

Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{ab} = v_{15\Omega}$ *directly!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_{Th} *gives,* $\frac{v_{Th}+4v_x-v_x}{10} + \frac{v_{Th}}{15} = 0$

It's clear that, $\frac{v_x}{5} = \frac{v_T+4v_x-v_x}{10}$

Rearranging the equations gives,

CONT...

$$v_{Th}\left(\frac{1}{10}+\frac{1}{15}\right)+v_x\left(\frac{3}{10}\right)=0$$

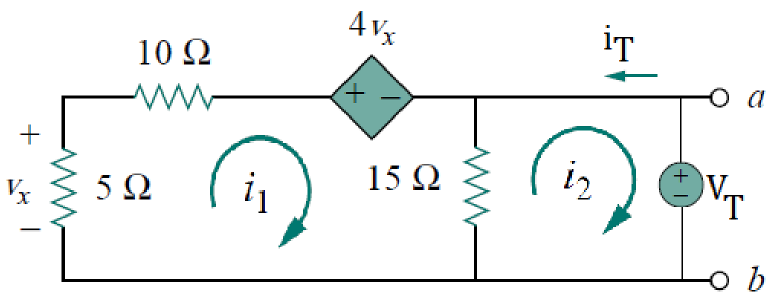
$$v_{Th}\left(-\frac{1}{10}\right)+v_x\left(\frac{1}{5}-\frac{3}{10}\right)=0$$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow\boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow\boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix}\frac{1}{6} & \frac{3}{10}\\ -\frac{1}{10} & -\frac{1}{10}\end{bmatrix}\begin{bmatrix}v_{Th}\\ v_x\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}\Rightarrow\begin{bmatrix}v_{Th}\\ v_x\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

This implies that, $v_{Th}=\mathbf{0V}$

Now in order to find the R_{Th}*, let redraw the circuit with test a test source and mesh currents!*



Assume that direction of voltage drops signed positive!

By KVL for each mesh gives,

$$5i_1+10i_1+4v_x+15(i_1-i_2)=0$$

$$v_T+15(i_2-i_1)=0$$

It's clear that, $5i_1=-v_x$

Rearranging the equations with substitution gives,

$$10i_1-15i_2+0v_T=0\Rightarrow 10i_1=\mathbf{15}\boldsymbol{i_2}$$

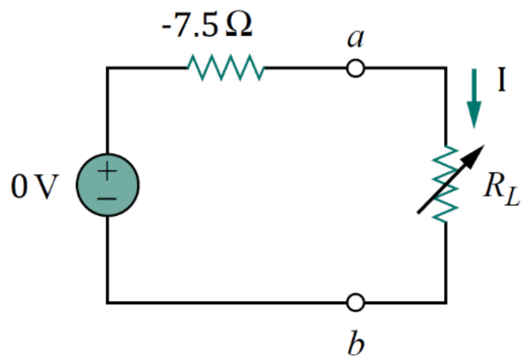
$$v_T=15i_1-15i_2\Rightarrow v_T=\mathbf{7.5}\boldsymbol{i_2}$$

It's clear-cut that, $i_T=-i_2$

It's obvious that, $R_{Th}=\frac{v_T}{i_T}=-\mathbf{7.5\Omega}$

So that the Thevenin equivalent circuit is the given below!

CONT...



4.11

Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{ab} = v_{6\Omega}$ *directly!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

Let denote v_{4A} *as* v_1*!*

By KCL for node voltages gives,

$$\frac{v_1 - 15}{3} - 4 + \frac{v_1 - v_{Th}}{3} = 0$$

$$\frac{v_{Th} - v_1}{3} + \frac{v_{Th}}{6} = 0$$

Rearranging the equations gives,

$$v_1\left(\frac{1}{3} + \frac{1}{3}\right) + v_{Th}\left(-\frac{1}{3}\right) = 9$$

$$v_1\left(-\frac{1}{3}\right) + v_{Th}\left(\frac{1}{3} + \frac{1}{6}\right) = 0$$

As it known, $\boldsymbol{Ax = b \Rightarrow A^{-1}Ax = A^{-1}b \Rightarrow x = A^{-1}b}$

$$\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_{Th} \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_{Th} \end{bmatrix} = \begin{bmatrix} 20.25 \\ 13.5 \end{bmatrix}$$

This implies that, $v_{Th} = \mathbf{13.5V}$

Let create a short-circuit for the terminals a and b in order to find short-circuit current i_{sc}*!*

This implies that, voltage drop across the 6Ω *resistor becomes zero!*

CONT...

Let denote v_{4A} *as* v_1*!*

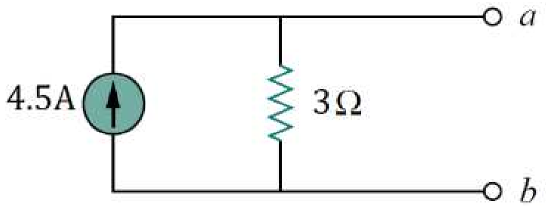
By KCL for node voltage v_1 *gives,* $\frac{v_1-15}{3}-4+\frac{v_1}{3}=0$

This implies, $v_1=\mathbf{13.5V}$

It's clear that, $i_{sc}=i_{3\Omega}=\frac{13.5-0}{3}=\mathbf{4.5A}$

As it known, $R_N=R_{Th}=\frac{v_T}{i_{sc}}=\frac{13.5}{4.5}=\mathbf{3\Omega}$ *and* $I_N=i_{sc}=\mathbf{4.5A}$

So that, the Norton equivalent circuit is the given below!



4.12

Because of we are trying to find Thevenin equivalent, $v_{Th}=v_{ab}=v_x$ *directly!*

By Ohm's Law, $v=i\cdot R\Rightarrow i=\frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i=\frac{v_{higher}-v_{lower}}{R}$

Let denote currents leaving a node positive!

Let denote v_{10A} *as* v_1*!*

It's obvious that dependent voltage source connects two non-reference nodes!

This implies that, source creates a supernode!

By KCL for supernode gives, $\frac{v_1}{6}-10+\frac{v_x}{2}=0$

It's clear that, $v_1-v_x=2v_x$

Rearranging the equations gives,

$$v_1\left(\frac{1}{6}\right)+v_x\left(\frac{1}{2}\right)=\mathbf{10}$$

$$v_1-3v_x=0$$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow\boldsymbol{A^{-1}Ax}=\boldsymbol{A^{-1}b}\Rightarrow\boldsymbol{x}=\boldsymbol{A^{-1}b}$

$$\begin{bmatrix}\frac{1}{6} & \frac{1}{2}\\ 1 & -3\end{bmatrix}\begin{bmatrix}v_1\\ v_x\end{bmatrix}=\begin{bmatrix}10\\ 0\end{bmatrix}\Rightarrow\begin{bmatrix}v_1\\ v_x\end{bmatrix}=\begin{bmatrix}30\\ 10\end{bmatrix}$$

CONT...

This implies that, $v_x = v_{Th} = \mathbf{10V}$

Let create a short-circuit for the terminals a and b in order to find short-circuit current i_{sc}*!*

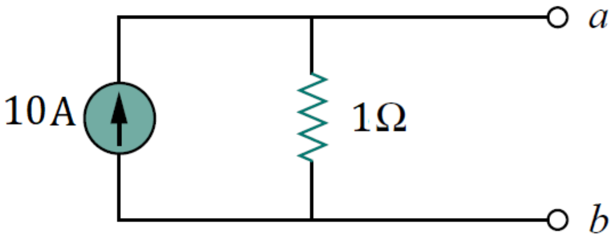
This implies that, voltage drop across the 2Ω *resistor becomes zero!*

So that, voltage drop across the dependent voltage source becomes zero!

This implies that, $i_{sc} = 10A$

As it known, $R_N = R_{Th} = \frac{v_{Th}}{i_{sc}} = 1\Omega$ *and* $I_N = i_s = \mathbf{10A}$

So that, the Norton equivalent circuit is the given below!



4.13

The load resistor will be ignored in order to find the open-circuit voltage!

Because of we are trying to find Thevenin equivalent, v_{Th} *is equal to open-circuit voltage directly!*

This implies that, voltage drop across the 4Ω *resistor becomes zero!*

So that the node voltage for the middle node is equal to v_{Th}*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for upper middle node gives, $\frac{v_T - 9}{2} + v_T - 3v_x = 0$

It's clear that, $v_x = 9 - v_{Th}$

Rearranging the equations gives,

$v_T\left(\frac{1}{2} + 1\right) - 3v_x = \frac{9}{2}$

$v_T + v_x = 9$

As it known, $\mathbf{Ax} = \mathbf{b} \Rightarrow \mathbf{A}^{-1}\mathbf{Ax} = \mathbf{A}^{-1}\mathbf{b} \Rightarrow \mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$

CONT...

$$\begin{bmatrix} \frac{3}{2} & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} v_{Th} \\ v_x \end{bmatrix} = \begin{bmatrix} \frac{9}{2} \\ 9 \end{bmatrix} \Rightarrow \begin{bmatrix} v_{Th} \\ v_x \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

This implies that, $v_{Th} = \mathbf{7V}$

Let create a short-circuit for the terminals a and b in order to find short-circuit current i_{sc}*!*

Let denote the upper middle node voltage as v_1*!*

By KCL for node v_1 *gives,* $\frac{v_1-9}{2} + v_1 - 3v_x + \frac{v_1}{4} = 0$

It's clear that, $v_x = 9 - v_1$

Rearranging the equations gives,

$v_1\left(\frac{1}{2} + 1 + \frac{1}{4}\right) - 3v_x = \frac{9}{2}$

$v_1 + v_x = 9$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

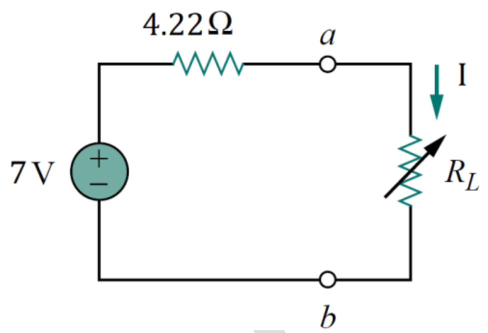
$$\begin{bmatrix} \frac{7}{4} & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_x \end{bmatrix} = \begin{bmatrix} \frac{9}{2} \\ 9 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_x \end{bmatrix} = \begin{bmatrix} 6.632 \\ 2.368 \end{bmatrix}$$

This implies that, $v_1 = 6.632V$

It's clear that, $i_{sc} = \frac{v_1}{4} = 1.658A$

As it known, $R_N = R_{Th} = \frac{v_{Th}}{i_{sc}} = 4.22\Omega$ *and* $I_N = i_{sc} = \mathbf{1.658A}$

So that the Thevenin equivalent is given below!



Here 4.22Ω *corresponds to* R_{Th}*!*

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i = i^2 \cdot R$

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections!*

This implies that, R_{eq} *for* R_{Th} *and* R_L *becomes* $R_{eq} = R_{Th} + R_L$

Now, $p = \left(\frac{\boldsymbol{v_{Th}}}{\boldsymbol{R_{Th}+R_L}}\right)^{\mathbf{2}} \cdot \boldsymbol{R_L}$

CONT...

In order to find the maximum power that transferred to load, we must differentiate the equation given above!

So that, $\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right] = V_{Th}^2 \left[\frac{R_{Th}-R_L}{(R_{Th}+R_2)^3}\right] = 0$

This implies that, the maximum power is transferred to the load when the load resistance equals to the Thevenin resistance!($R_L = R_{Th}$ *!)*

So that, $p_{max} = \left(\frac{v_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L = \left(\frac{v_{Th}}{2R_{Th}}\right)^2 \cdot R_{Th} = \frac{v_{Th}^2}{4R_{Th}} = \mathbf{2.90W}$

4.15

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i = i^2 \cdot R$

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections!*

This implies that, R_{eq} *for* R_{Th} *and* R_L *becomes* $R_{eq} = R_{Th} + R_L$

Now, $p = \left(\frac{v_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L$

In order to find the maximum power that transferred to load, we must differentiate the equation given above!

So that, $\frac{dp}{dR_L} = v_{Th}^2 \left[\frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right] = v_{Th}^2 \left[\frac{R_{Th}-R_L}{(R_{Th}+R_2)^3}\right] = 0$

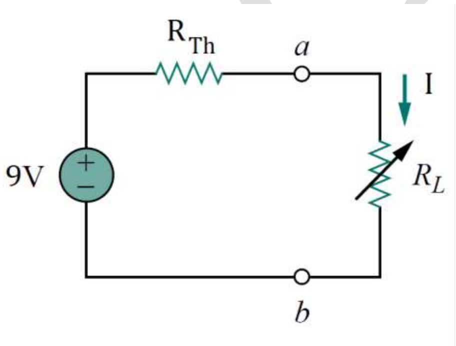
This implies that, the maximum power is transferred to the load when the load resistance equals to the Thevenin resistance!($R_L = R_{Th}$ *!)*

It's clear that, v_{Th} *is equal to open-circuit voltage* $1V$ *and* $R_{Th} = \mathbf{2k\Omega}$ ***!***

This implies, $p_{max} = \left(\frac{v_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L = \left(\frac{v_{Th}}{2R_{Th}}\right)^2 \cdot R_{Th} = \frac{v_{Th}^2}{4R_{Th}} = 0.125W = \mathbf{125mW}$

4.16

Let redraw the circuit with the load resistor!



Here R_L *corresponds to* 20Ω *resistance!*

CONT...

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections!*

This implies that, R_{eq} *for* R_{Th} *and* R_L *becomes* $R_{eq} = R_{Th} + R_L$

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

By substitution, $v_{R_L} = \frac{20}{R_{eq}} 9V = 8V \Rightarrow R_{eq} = 22.5\Omega \Rightarrow R_{Th} = 2.5\Omega$

Now, for the circuit given above, R_{Th} *corresponds to* 2.5Ω*!*

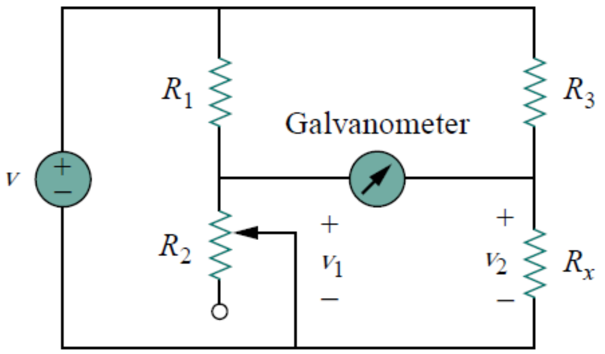
As it known, new value of R_L *is* 10Ω

This implies that, R_{eq} *for* R_{Th} *and* R_L *becomes* $R_{eq} = R_{Th} + R_L = \mathbf{12.5\Omega}$

By substitution, $v_{R_L} = \frac{10}{12.5} 9V = \mathbf{7.2V}$

4.17

Let redraw the circuit first!



By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

Because of the bridge is balanced $v_1 = v_2$*!*

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

This implies that, $v_1 = \frac{R_2}{R_1+R_2} v = v_2 = \frac{R_x}{R_x+R_3} v$

It's clear that, no current flows through the galvanometer when, $\frac{R_2}{R_1+R_2} = \frac{R_x}{R_x+R_3} \Rightarrow R_2R_3 = R_1R_x$

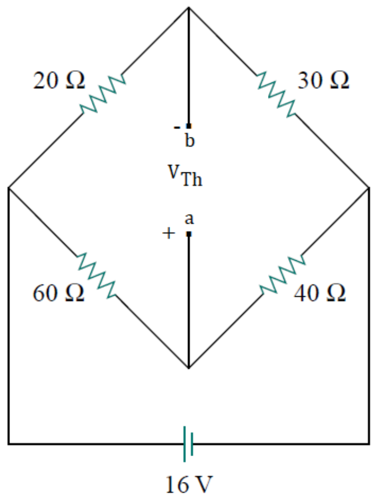
This implies, $R_x = \frac{R_3}{R_1} R_2$

By substitution, $R_x = \mathbf{3.2k\Omega}$

4.18

Let redraw the circuit as open-circuit!

CONT...



By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

It's clear that, 60Ω *and* 40Ω *are connected in series so that* R_{eq} *for them is* $60\Omega + 40\Omega = 100\Omega$

Also, 20Ω *and* 30Ω *are connected in series so that* R_{eq} *for them is* $20\Omega + 30\Omega = 50\Omega$

It's obvious that, $v_{50\Omega} = v_{100\Omega} = 16V$ *because they are both connected to the same pair of terminals!(if there is any confusion you*

can verify it using Kirchhoff's Voltage Law!)

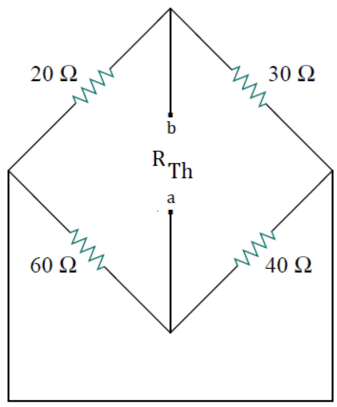
Because of we are trying to find Thevenin equivalent, v_{Th} *is equal to open-circuit voltage directly!*

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

This implies that, $v_{40\Omega} = \frac{40}{100} 16V = 6.4V$ *and* $v_{30\Omega} = \frac{30}{50} 16V = \mathbf{9.6V}$

It's clear that, $v_{Th} = v_{oc} = 6.4V - 9.6V = \mathbf{-3.2V}$

Let redraw the circuit in order to find Thevenin resistance!



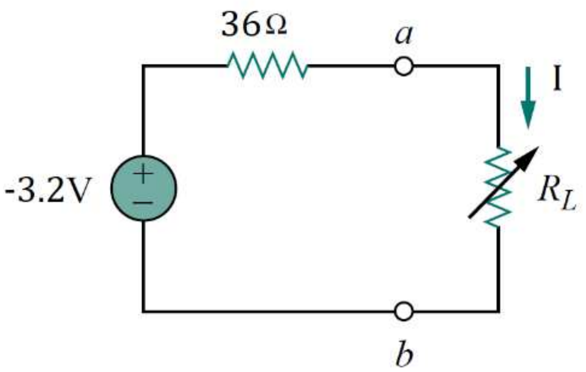
CONT...

It's clear that, 60Ω *and* 40Ω *are connected int parallel so that* R_{eq} *for them is,* $\frac{1}{60}+\frac{1}{40}=\frac{1}{R_{eq}} \Rightarrow R_{eq}=24\Omega$

Now, 20Ω *and* 30Ω *are connected int parallel so that* R_{eq} *for them is,* $\frac{1}{20}+\frac{1}{30}=\frac{1}{R_{eq}} \Rightarrow R_{eq}=12\Omega$

Finally, 24Ω *and* 12Ω *are connected int series so that* R_{eq} *for them is* $24\Omega+12\Omega=\mathbf{36\Omega}$

So that the Thevenin equivalent is given below!



Here R_L *corresponds to* 14Ω*!*

It's clear that, I *is equal to the current that flows through the galvanometer!*

Assume that direction of voltage drops signed positive and direction of current I *through the loop is clockwise direction!*

By KVL for closed path gives, $3.2+36I+14I=0 \Rightarrow I=-0.064A=\mathbf{-64mA}$

Note that, negative sign indicates that the current flows in the direction opposite to the one assumed, that is, from terminal b to a!

Chapter 4-Problems

4.1

As it known, $R_{eq}=\sum_{n=1}^{\infty} R_n$ *for the series connections!*

By Ohm's Law, $v=i\cdot R \Rightarrow i=\frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i=\frac{v_{higher}-v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for upper middle node gives, $v_1-1+\frac{v_1}{8}+\frac{v_1}{8}=0 \Rightarrow v_1=\mathbf{0.8V}$

It's clear that, $i_0=\frac{v_1}{8}=\mathbf{0.1A}$

CONT...

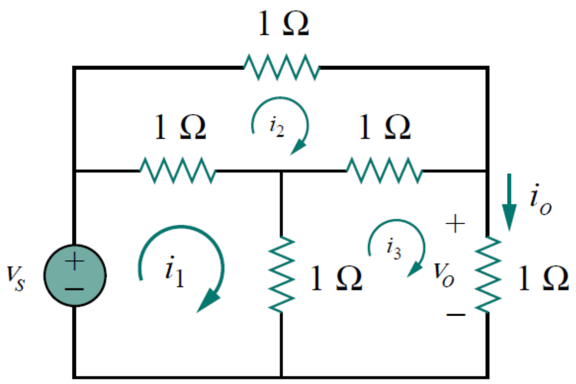
By the linearity property, $kiR = kv$

This implies that, if the input voltage becomes $10V$*,* k *becomes* $10!$

This implies that, new value of i_0 *is,* $ki_0 = \mathbf{1A}$

4.3

(a) Let redraw the circuit with mesh currents!



By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

Assume that direction of voltage drops signed positive!

By KVL for each mesh gives,

$-v_s + i_1 - i_2 + i_1 - i_3 = 0$

$i_2 + i_2 - i_3 + i_2 - i_1 = 0$

$i_3 - i_2 + i_3 + i_3 - i_1 = 0$

Rearranging the equations with substitution gives,

$2i_1 - i_2 - i_3 = 1$

$-i_1 + 3i_2 - i_3 = 0$

$-i_1 - i_2 + 3i_3 = 0$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix}$$

This implies that, $i_1 = 1A$*,* $i_2 = 0.5A$ *and* $i_3 = 0.5A$

It's clear that, $i_0 = i_3 = \mathbf{0.5A}$ $v_0 = i_0 \Rightarrow v_0 = \mathbf{0.5V}$

(b) By the linearity property, $kiR = kv$

This implies that, if the source voltage becomes $10V$*,* k *becomes* $10!$

So that, new value of v_0 *becomes* $kv_0 = \mathbf{5V}$ *and* $i_0 = \mathbf{5A}$

(c) By KVL for each mesh gives,

By KVL for each mesh gives,

$-v_s + 10(i_1 - i_2) + 10(i_1 - i_3) = 0$

$10i_2 + 10(i_2 - i_3) + 10(i_2 - i_1) = 0$

$10(i_3 - i_2) + 10i_3 + 10(i_3 - i_1 = 0$

Rearranging the equations with substitution gives,

$20i_1 - 10i_2 - 10i_3 = 10$

$-10i_1 + 30i_2 - 10i_3 = 0$

$-10i_1 - 10i_2 + 30i_3 = 0$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 20 & -10 & -10 \\ -10 & 30 & -10 \\ -10 & -20 & 30 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix}$$

This implies that, $i_1 = 1A$, $i_2 = 0.5A$ *and* $i_3 = 0.5A$

It's clear that, $i_0 = i_3 = \mathbf{0.5A}$ *and* $v_0 = i_0 \Rightarrow v_0 = \mathbf{0.5V}$

4.5

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections!*

Let denote the voltage drop across the first 6Ω *resistor as* v_1*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive and source voltage as v_{source}*!*

By KCL for node voltages gives,

$$\frac{v_1 - v_{source}}{2} + \frac{v_1}{6} + \frac{v_1 - 1}{3} = 0$$

$$\frac{1 - v_1}{3} + \frac{1}{6} + \frac{1}{6} = 0$$

It's clear that, $v_1 = 2V$

By using substitution, $v_{source} = 3.333V$

By the linearity property, $kiR = kv$

CONT...

This implies that, if the source voltage becomes $15V$*,* k *becomes* 4.5*!*

So that, new value of v_0 *becomes* $kv_0 = \mathbf{4.5V}$

4.7

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections!*

Let denote the voltage drop across the first 3Ω *resistor as* v_1*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive and source voltage as v_{source}*!*

By KCL for node voltage gives, $v_1 - v_{source} + \frac{v_1}{3} + \frac{v_1 - 1}{4} = 0$

It's clear that, $\frac{v_1 - 1}{4} = \frac{1}{2} \Rightarrow v_1 = 3V$

By using substitution, $v_{source} = 4.5V$

By the linearity property, $kiR = kv$

This implies that, if the source voltage becomes $4V$*,* k *becomes* 0.8889*!*

So that, new value of V_0 *becomes* $kV_0 = 0.8889V = \mathbf{888.9mV}$

4.9

Let $v_0 = v_1 + v_2$ *where* v_1 *and* v_2 *are the contributions due to the* $6A$ *current and* $18V$ *voltage sources, respectively!*

To obtain v_1 *set voltage source as zero!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

It's clear that, 2Ω *and* 2Ω *are connected in parallel so that* R_{eq} *for them is* $\frac{1}{2} + \frac{1}{2} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 1\Omega$

Now, 1Ω *and* 1Ω *are connected int series so that* R_{eq} *for them is* $1\Omega + 1\Omega = 2\Omega$

By Current Division equation, $i_x = \frac{R_{eq}}{R_x} i_{source}$

It's clear that, 2Ω *and* 4Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{2} + \frac{1}{4} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 1.333\Omega$

Let denote the current flows through the v_0 *as* i_0

$\boldsymbol{CONT...}$

This implies that, $i_0 = \frac{1.333}{2} 6A = \mathbf{3.999A}$

So that, $v_1 = i_0 = 4$

To obtain v_2 *set current source as zero!*

It's clear that, 2Ω *and* 2Ω *are connected in parallel so that* R_{eq} *for them is* $\frac{1}{2} + \frac{1}{2} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 1\Omega$

Now, 1Ω, 1Ω *and* 4Ω *are connected int series so that* R_{eq} *for them is* $1\Omega + 1\Omega + 4\Omega = 6\Omega$

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

By substitution, $v_2 = \frac{1}{6} 18V = 3V$

This implies, $v_0 = v_1 + v_2 = 3.999V + 3V = \mathbf{6.999V}$

4.11

Let denote $v_{40\Omega}$ *as* v_a, v_{4i_0} *as* v_b *and lower node as reference node!*

Let $v_0 = v_1 + v_2$ *where* v_1 *and* v_2 *are the contributions due to the* $6A$ *current and* $30V$ *voltage sources, respectively!*

To obtain v_1 *set voltage source as zero!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives,

$$-6 + \frac{v_a}{40} + \frac{v_a - v_b}{10} = 0$$

$$\frac{v_b - v_a}{10} - 4i_1 + \frac{v_b}{20} = 0$$

It's clear that, $i_1 = \frac{v_a - v_b}{10}$

Rearranging the equations gives,

$$v_a\left(\frac{1}{40} + \frac{1}{10}\right) + v_b\left(-\frac{1}{10}\right) + 0i_1 = 6$$

$$v_a\left(-\frac{1}{10}\right) + v_b\left(\frac{1}{10} + \frac{1}{20}\right) - 4i_1 = 0$$

$$v_a\left(-\frac{1}{10}\right) + v_b\left(\frac{1}{10}\right) + i_1 = 0$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

CONT...

$$\begin{bmatrix} \frac{1}{8} & -\frac{1}{10} & 0 \\ -\frac{1}{10} & \frac{3}{20} & -4 \\ -\frac{1}{10} & \frac{1}{10} & 1 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ i_1 \end{bmatrix} = \begin{bmatrix} 6 \\ \frac{3}{2} \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_a \\ v_b \\ i_1 \end{bmatrix} = \begin{bmatrix} 176 \\ 160 \\ 1.6 \end{bmatrix}$$

This implies that, $i_1 = 1.6A$

It's clear that, $v_1 = 10i_1 \Rightarrow v_1 = 16V$

To obtain v_2 *set current source as zero!*

By using same process rearranged equations are given below!

$$v_a\left(\frac{1}{40}+\frac{1}{10}\right)+v_b\left(-\frac{1}{10}\right)+0i_2 = 0$$

$$v_a\left(-\frac{1}{10}\right)+v_b\left(\frac{1}{10}+\frac{1}{20}\right)-4i_2 = -\frac{3}{2}$$

$$v_a\left(-\frac{1}{10}\right)+v_b\left(\frac{1}{10}\right)+i_2 = 0$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} \frac{1}{8} & -\frac{1}{10} & 0 \\ -\frac{1}{10} & \frac{3}{20} & -4 \\ -\frac{1}{10} & \frac{1}{10} & 1 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{3}{2} \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_a \\ v_b \\ i_1 \end{bmatrix} = \begin{bmatrix} -8 \\ -10 \\ 0.2 \end{bmatrix}$$

This implies that, $i_1 = 0.2A$

It's clear that, $v_2 = 10i_2 \Rightarrow v_1 = 2V$

So that, $v_0 = v_1 + v_2 = 16V + 2V = \mathbf{18V}$

This implies, $i_0 = \frac{v_0}{10} = \mathbf{1.8A}$

4.13

Let $v_0 = v_1 + v_2$ *where* v_1 *and* v_2 *are the contributions due to the independent current sources and* $12V$ *voltage source, respectively!*

To obtain v_1 *set voltage source as zero!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

Let denote $v_{10\Omega}$ *as* v_a *!*

CONT...

By KCL for node voltage gives,

$$-2+4+\frac{v_a-v_1}{8}+\frac{v_a}{10}=0$$

$$-4+\frac{v_1}{5}+\frac{v_1-v_a}{8}=0$$

Rearranging the equations gives,

$$v_1\left(-\frac{1}{8}\right)+v_a\left(\frac{1}{8}+\frac{1}{10}\right)=-2$$

$$v_1\left(\frac{1}{5}+\frac{1}{8}\right)+v_a\left(-\frac{1}{8}\right)=4$$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow\boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow\boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix}-\frac{1}{8} & \frac{9}{40}\\ \frac{13}{40} & -\frac{1}{8}\end{bmatrix}\begin{bmatrix}v_1\\ v_a\end{bmatrix}=\begin{bmatrix}-2\\ 4\end{bmatrix}\Rightarrow\begin{bmatrix}v_1\\ v_a\end{bmatrix}=\begin{bmatrix}11.304\\ -2.609\end{bmatrix}$$

This implies that, $v_1=11.304V$

To obtain v_2 *set current sources as zero!*

Let denote the current that flows through the circuit as i_s*!*

Assume that direction of voltage drops signed positive!

By KVL for closed path gives, $10i_s+8i_s+12+v_2=0$

It's clear that, $v_2=5i_s$

Rearranging the equations gives,

$$18i_s+v_2=-12$$

$$-5i_s+v_2=0$$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow\boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow\boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix}18 & 1\\ -5 & 1\end{bmatrix}\begin{bmatrix}i_s\\ v_2\end{bmatrix}=\begin{bmatrix}-12\\ 0\end{bmatrix}\Rightarrow\begin{bmatrix}i_s\\ v_2\end{bmatrix}=\begin{bmatrix}-0.5217\\ -2.609\end{bmatrix}$$

This implies that, $v_2=\boldsymbol{-2.609V}$

So that, $v_0=v_1+v_2=11.304V-2.609V=\boldsymbol{8.695V}$

4.15

Let $i=i_1+i_2$ *where* i_1 *and* i_2 *are the contributions due to the* $0.2A$ *source and independent voltage sources, respectively!*

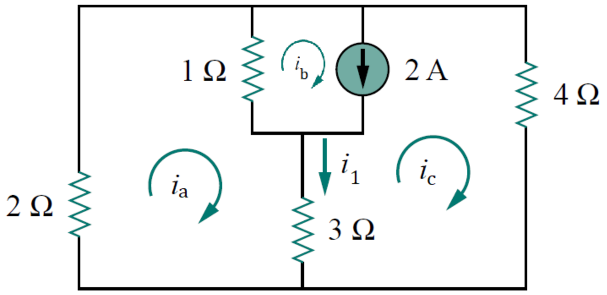
To obtain v_1 *set voltage sources as zero!*

By Ohm's Law, $v=i\cdot R\Rightarrow i=\frac{v}{R}$

CONT...

Assume that direction of voltage drops signed positive

Let redraw the circuit with mesh currents!



It's obvious that, independent current source causes a supermesh!(Two meshes have a current source in common!)

By KVL for supermesh gives, $i_b - i_a + 4i_c + 3(i_c - i_a) = 0$

By KVL for mesh i_a *gives,*

$2i_a + i_a - i_b + 3(i_a - i_c) = 0$

It's clear that, $i_b - i_c = 2$

Rearranging the equations gives,

$-4i_a + i_b + 7i_c = 0$

$6i_a - i_b - 3i_c = 0$

$0i_a + i_b - i_c = 2$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

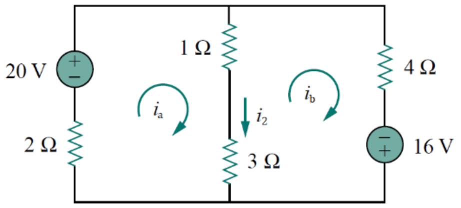
$$\begin{bmatrix} -4 & 1 & 7 \\ 6 & -1 & -3 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 0.25 \\ 1.875 \\ -0.125 \end{bmatrix}$$

This implies that, $i_a = 0.25A,\ i_b = 1.875A$ *and* $i_c = -0.125A$

It's clear that, $i_1 = i_a - i_c = 0.375A = 375mA$

To obtain v_2 *set current source as zero!*

Now let redraw the circuit with mesh currents!



By KVL for each mesh gives,

CONT...

$2i_a - 20 + i_a - i_b + 3(i_a - i_b) = 0$

$4i_b - 16 + 3(i_b - i_a) + i_b - i_a = 0$

$6i_a - 4i_b = 20$

$-4i_a + 8i_b = 16$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$\begin{bmatrix} 6 & -4 \\ -4 & 8 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 20 \\ 16 \end{bmatrix} \Rightarrow \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 7 \\ 5.5 \end{bmatrix}$

This implies that, $i_a = 7A$ *and* $i_b = 5.5A$

It's clear that, $i_2 = i_a - i_b = 1.5A$

So that, $i = i_1 + i_2 = 0.375A + 1.5A = \mathbf{1.875A}$

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i = i^2 \cdot R$

This implies, $p_{3\Omega} = 3i^2 = \mathbf{10.55W}$

4.17

Let $v_x = v_1 + v_2$ *where* v_1 *and* v_2 *are the contributions due to the* $6A$ *source and independent voltage sources, respectively!*

To obtain v_1 *set voltage sources as zero!*

Let denote $v_{60\Omega}$ *as* v_a *and* v_{6A} *as* v_b*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives,

$\frac{v_a}{30} + \frac{v_a}{60} + \frac{v_a - v_b}{10} = 0$

$\frac{v_b - v_a}{10} - 6 + \frac{v_b}{30} + \frac{v_b}{20} = 0$

Rearranging the equations gives,

$v_a\left(\frac{1}{30} + \frac{1}{60} + \frac{1}{10}\right) + v_b\left(-\frac{1}{10}\right) = 0$

$v_a\left(-\frac{1}{10}\right) + v_b\left(\frac{1}{10} + \frac{1}{30} + \frac{1}{20}\right) = 6$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

CONT...

$$\begin{bmatrix} \frac{3}{20} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{11}{60} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} 34.286 \\ 51.429 \end{bmatrix}$$

This implies that, $v_a = 34.286V$, $v_b = 51.429V$ *and* $v_1 = v_a - v_b = -17.143V$

To obtain v_2 *set current source as zero!*

Let denote $v_{60\Omega}$ *as* v_a *and* $v_{30\Omega}$ *as* v_b*!*

By KCL for node voltages gives,

$$\frac{v_a - 90}{30} + \frac{v_a}{60} + \frac{v_a - v_b}{10} = 0$$

$$\frac{v_b - v_a}{10} + \frac{v_b}{30} + \frac{v_b - 40}{20} = 0$$

Rearranging the equations gives,

$$v_a\left(\frac{1}{30} + \frac{1}{60} + \frac{1}{10}\right) + v_b\left(-\frac{1}{10}\right) = 3$$

$$v_a\left(-\frac{1}{10}\right) + v_b\left(\frac{1}{10} + \frac{1}{30} + \frac{1}{20}\right) = 2$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} \frac{3}{20} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{11}{60} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} 42.857 \\ 34.286 \end{bmatrix}$$

This implies that, $v_a = 42.857V$, $v_b = 34.286V$ *and* $v_2 = v_a - v_b = \mathbf{8.571V}$

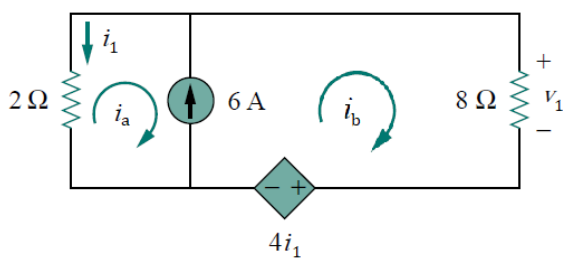
So that, $v_x = v_1 + v_2 = -17.143V + 8.571V = \mathbf{-8.572V}$

4.19

Let $v_x = v_1 + v_2$ *where* v_1 *and* v_2 *are the contributions due to the* $6A$ *and* $4A$ *sources, respectively!*

To obtain v_1 *set* $4A$ *source as zero!*

Let redraw the circuit with mesh currents!



By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

CONT...

Assume that direction of voltage drops signed positive!

It's obvious that, independent current source causes a supermesh!(Two meshes have a current source in common!)

By KVL for supermesh gives, $2i_a + 8i_b + 4i_1 = 0$

It's clear that, $i_1 = -i_a$ *and* $i_b - i_a = 6$

Rearranging the equations gives,

$2i_a + 8i_b + 4i_1 = 0$

$i_a + 0i_b + i_1 = 0$

$-i_a + i_b + 0i_1 = 6$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

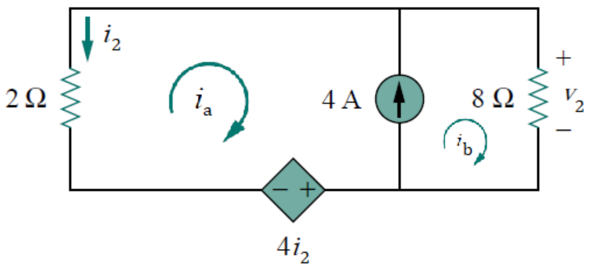
$$\begin{bmatrix} 2 & 8 & 4 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} i_a \\ i_b \\ i_1 \end{bmatrix} = \begin{bmatrix} -8 \\ -2 \\ 8 \end{bmatrix}$$

This implies that, $i_a = -8A,\ i_b = -2A$ *and* $i_1 = 8A$

It's clear that, $v_1 = 8i_b = \boldsymbol{-16V}$

To obtain v_2 *set* $6A$ *source as zero!*

Let redraw the circuit with mesh currents!



It's obvious that, independent current source causes a supermesh!(Two meshes have a current source in common!)

By KVL for supermesh gives, $2i_a + 8i_b + 4i_1 = 0$

It's clear that, $i_2 = -i_a$ *and* $i_b - i_a = 4$

Rearranging the equations gives,

$2i_a + 8i_b + 4i_1 = 0$

$i_a + 0i_b + i_2 = 0$

$-i_a + i_b + 0i_2 = 4$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

CONT...

$$\begin{bmatrix} 2 & 8 & 4 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} i_a \\ i_b \\ i_1 \end{bmatrix} = \begin{bmatrix} -5.333 \\ -1.333 \\ 5.333 \end{bmatrix}$$

This implies that, $i_a = -5.333A$, $i_b = -1.333A$ *and* $i_1 = 5.333A$

It's clear that, $v_2 = 8i_b = -10.66V$

So that, $v_x = v_1 + v_2 = -16V - 10.66V = \mathbf{-26.66V}$

4.23

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

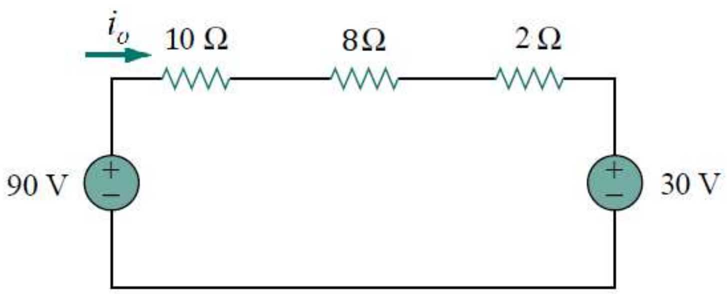
It's obvious that, $45V$ *source and* 3Ω *resistor are connected in series so that, they can be replaced with* $15A$ *source and* 3Ω *resistor connected in parallel!(Note that direction must be upward!)*

It's clear that, 6Ω *and* 3Ω *are connected in parallel so* R_{eq} *for them is* $\frac{1}{6} + \frac{1}{3} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 2\Omega$

Now, $15A$ *source and* 2Ω *resistor are connected in parallel so that, they can be replaced with* $30V$ *source and* 2Ω *resistor connected in series!*

It's obvious that, $9A$ *source and* 10Ω *resistor are connected in parallel so that, they can be replaced with* $90V$ *source and* 10Ω *resistor connected in series!*

So that the circuit becomes,



Assume that direction of voltage drops signed positive!

By KVL for closed path gives, $-90 + 10i_0 + 8i_0 + 2i_0 + 30 = 0 \Rightarrow i_0 = \mathbf{3A}$

This implies, $v_{8\Omega} = 8i_0 = \mathbf{24V}$

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i$

This implies, $p_{8\Omega} = v_{8\Omega} \cdot i_{8\Omega} = \mathbf{72W}$

CONT...

4.25

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

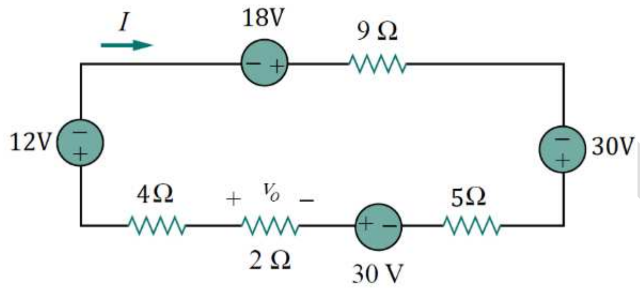
By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

It's obvious that, $6A$ *source and* 5Ω *resistor are connected in parallel so that, they can be replaced with* $30V$ *source and* 5Ω *resistor connected in series!(Note that direction of the voltage drop must be upward!)*

Now, $2A$ *source and* 9Ω *resistor are connected in parallel so that, they can be replaced with* $18V$ *source and* 9Ω *resistor connected in series!(Note that direction of the voltage rise must be to the right!)*

It's clear that, $3A$ *source and* 4Ω *resistor are connected in parallel so that, they can be replaced with* $12V$ *source and* 4Ω *resistor connected in series!(Note that direction of the voltage drop must be upward!)*

So that the circuit becomes,



Assume that direction of voltage drops signed positive!

By KVL for closed path gives, $12 - 18 + 9I - 30 + 5I - 30 + 2I + 4I = 0 \Rightarrow I = \mathbf{3.3A}$

This implies, $v_0 = -2I = \mathbf{-6.6V}$

4.27

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

It's obvious that, $40V$ *source and* 20Ω *resistor are connected in series so that, they can be replaced with* $2A$ *source and* 20Ω *resistor connected in parallel!(Note that direction must be upward!)*

Now, in order to create single source current, just add two independent current sources $8A + 2A = 10A$

CONT...

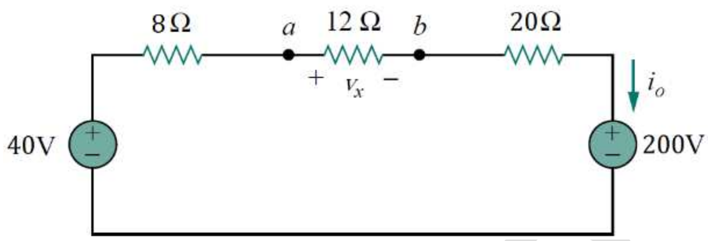
It's clear that, $10A$ *source and* 20Ω *resistor are connected in parallel so that, they can be replaced with* $200V$ *source and* 20Ω *resistor connected in series!(Note that direction of the voltage rise must be upward!)*

It's obvious that, $50V$ *source and* 10Ω *resistor are connected in series so that, they can be replaced with* $5A$ *source and* 10Ω *resistor connected in parallel!(Note that direction must be upward!)*

Now, 10Ω *and* 40Ω *are connected in parallel so that* R_{eq} *for them is* $\frac{1}{10}+\frac{1}{40}=\frac{1}{R_{eq}}\Rightarrow R_{eq}=8\Omega$

It's clear that, $5A$ *source and* 8Ω *resistor are connected in parallel so that, they can be replaced with* $40V$ *source and* 8Ω *resistor connected in series!(Note that direction of the voltage rise must be upward!)*

So that the circuit becomes,



Assume that direction of voltage drops signed positive!

By KVL for closed path gives, $-40+8i_0+12i_0+20i_0+200=0\Rightarrow i_0=\mathbf{-4A}$

This implies, $v_x=12i_0=\mathbf{-48V}$

4.29

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq}=\sum_{n=1}^{\infty}R_n$ *for the series connections and* $R_{eq}=\left(\sum_{n=1}^{\infty}\frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

By Ohm's Law, $v=i\cdot R\Rightarrow i=\frac{v}{R}$

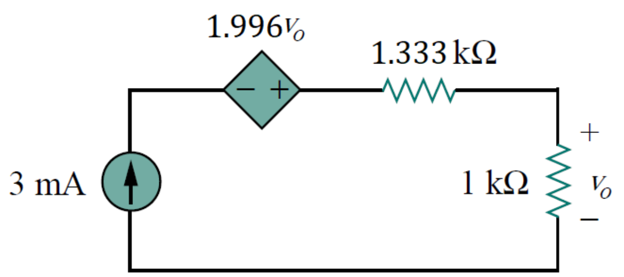
It's obvious that, dependent voltage source and $2k\Omega$ *resistor are connected in series so that, they can be replaced with* $\frac{3v_0}{2000}$ *current source and* $2k\Omega$ *resistor connected in parallel!(Note that direction must be to the right!)*

Now, $4k\Omega$ *and* $2k\Omega$ *are connected in parallel so that* R_{eq} *for them is* $\frac{1}{4}+\frac{1}{2}=\frac{1}{R_{eq}}\Rightarrow R_{eq}=1.333k\Omega$

It's clear that, $\frac{3v_0}{2000}$ *source and* $1.333k\Omega$ *resistor are connected in parallel so that, they can be replaced with* $1.996v_0$ *source and* $1.333k\Omega$ *resistor connected in series!(Note that direction of the voltage rise must be to the right!)*

So that the circuit becomes,

CONT...



It's clear that, $v_0 = (3 \times 10^{-3} A) \cdot (1 \times 10^{3} \Omega) = \boldsymbol{3V}$

4.31

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty} \frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

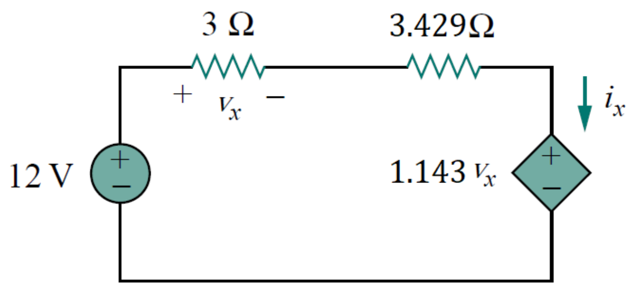
By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

It's obvious that, dependent voltage source and 6Ω *resistor are connected in series so that, they can be replaced with* $\frac{v_x}{3}$ *current source and* 6Ω *resistor connected in parallel!(Note that direction must be to the upward!)*

Now, 8Ω *and* 6Ω *are connected in parallel so that* R_{eq} *for them is* $\frac{1}{8} + \frac{1}{6} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 3.429\Omega$

It's clear that, $\frac{v_x}{3}$ *source and* 3.429Ω *resistor are connected in parallel so that, they can be replaced with* $1.143v_x$ *source and* 3.429Ω *resistor connected in series!(Note that direction of the voltage rise must be to the upward!)*

So that the circuit becomes,



Assume that direction of voltage drops signed positive!

By KVL for closed path gives, $-12 + 3i_x + 3.429i_x + 1.143v_x = 0$

It's clear that, $v_x = 3i_x$

Rearranging the equations gives,

CONT...

$6.429i_x + 1.143v_x = 12$

$-3i_x + v_x = 0$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$\begin{bmatrix} 6.429 & 1.143 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} i_x \\ v_x \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_x \\ v_x \end{bmatrix} = \begin{bmatrix} 1.217 \\ 3.652 \end{bmatrix}$

This implies that, $v_x = \mathbf{3.652V}$

4.33

(a) Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{12} = v_{4\Omega}$ *directly!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltage v_{Th} *gives,* $\frac{v_{Th}-20}{10} + \frac{v_{Th}}{40} = 0 \Rightarrow v_{Th} = 16V$

Now, let create a short-circuit for the terminals 1 and 2!

Let denote short-circuit current as i_{sc}*!*

This implies that, voltage drop across the 40Ω *resistor becomes zero!*

Assume that direction of voltage drops signed positive!

By KVL for closed path gives, $-20 + 10i_{sc} = 0 \Rightarrow i_{sc} = 2A$

As it known, $R_{Th} = \frac{v_{Th}}{i_{sc}} = 8\Omega$

(b) Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{12} = v_{30\Omega}$ *directly!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

By KCL for node voltage v_{Th} *gives,* $-2 + \frac{v_{Th}}{30} + \frac{v_{Th}-30}{60} = 0 \Rightarrow v_{Th} = 50V$

Now, let create a short-circuit for the terminals 1 and 2!

Let denote short-circuit current as i_{sc}*!*

This implies that, voltage drop across the 30Ω *resistor becomes zero!*

By KCL for the short-circuit node gives, $-2 + i_{sc} - \frac{30}{60} = 0 \Rightarrow i_{sc} = 2.5A$

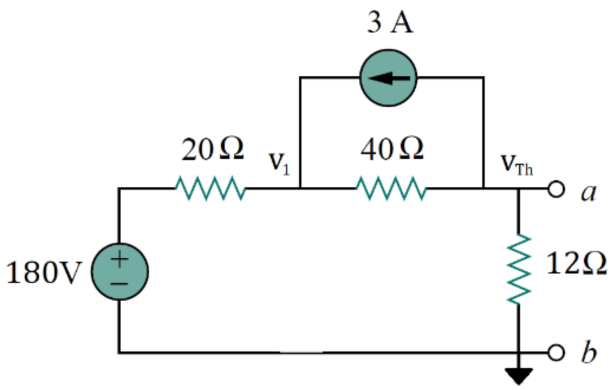
As it known, $R_{Th} = \frac{v_{Th}}{i_{sc}} = \mathbf{20\Omega}$

CONT...

4.37

Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{ab} = v_{12\Omega}$ *directly!*

Let redraw the circuit with node voltages and reference node!



By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives,

$$\frac{v_1 - 180}{20} - 3 + \frac{v_1 - v_{Th}}{40} = 0$$

$$3 + \frac{v_{Th} - v_1}{40} + \frac{v_{Th}}{12} = 0$$

Rearranging the equations gives,

$$v_1\left(\frac{1}{20} + \frac{1}{40}\right) + v_{Th}\left(-\frac{1}{40}\right) = 12$$

$$v_1\left(-\frac{1}{40}\right) + v_{Th}\left(\frac{1}{40} + \frac{1}{12}\right) = -3$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} \frac{3}{40} & -\frac{1}{40} \\ -\frac{1}{40} & \frac{13}{120} \end{bmatrix} \begin{bmatrix} v_1 \\ v_{Th} \end{bmatrix} = \begin{bmatrix} 12 \\ -3 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_{Th} \end{bmatrix} = \begin{bmatrix} 163.3 \\ 10 \end{bmatrix}$$

This implies that, $v_{Th} = 10V$

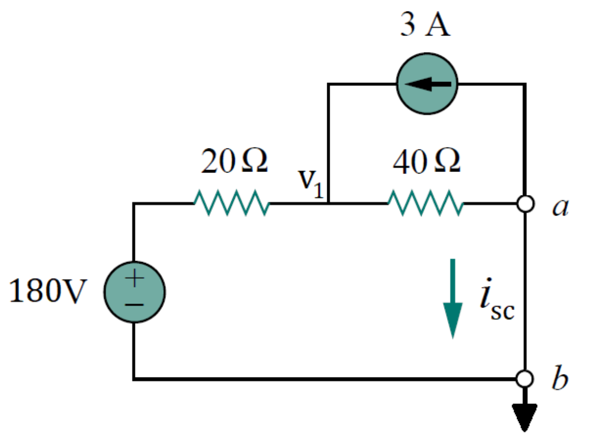
Now, let create a short-circuit for the terminals a and b!

Let denote short-circuit current as i_{sc}*!*

This implies that, voltage drop across the 12Ω *resistor becomes zero!*

CONT...

Let redraw the circuit with a node voltage and reference node!

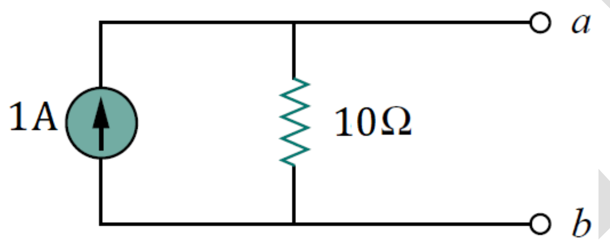


By KCL for node v_1 *gives,* $\frac{v_1-180}{20} - 3 + \frac{v_1}{40} = 0 \Rightarrow v_1 = \mathbf{160V}$

By KCL for node a gives, $3 - \frac{v_1}{40} + i_s = 0 \Rightarrow i_s = \mathbf{1A}$

As it known, $I_N = i_{sc}$ *and* $R_N = R_{Th} = \frac{v_{Th}}{i_{sc}} = \mathbf{10\Omega}$

So that the Norton equivalent circuit is given below!



4.39

Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{ab}$ *directly!*

Let denote the bottom node as reference node, first middle node as v_1 *and* $v_{5\Omega}$ *as* v_2*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives,

CONT...

$$-1+\frac{v_1-8}{10}+\frac{v_1-v_2}{10}=0$$

$$\frac{v_2-v_1}{10}+\frac{v_2}{5}+\frac{v_2-v_{Th}}{16}=0$$

$$1+\frac{v_{Th}-v_2}{16}=0$$

Rearranging the equations gives,

$$v_1\left(\frac{1}{10}+\frac{1}{10}\right)+v_2\left(-\frac{1}{10}\right)+0v_{Th}=\frac{9}{5}$$

$$v_1\left(-\frac{1}{10}\right)+v_2\left(\frac{1}{10}+\frac{1}{5}+\frac{1}{16}\right)+v_{Th}\left(-\frac{1}{16}\right)=0$$

$$0v_1+v_2\left(-\frac{1}{16}\right)+v_{Th}\left(\frac{1}{16}\right)=-1$$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow\boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow\boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix}\frac{1}{5} & -\frac{1}{10} & 0\\ -\frac{1}{10} & \frac{29}{80} & -\frac{1}{16}\\ 0 & -\frac{1}{16} & \frac{1}{16}\end{bmatrix}\begin{bmatrix}v_1\\ v_2\\ v_{Th}\end{bmatrix}=\begin{bmatrix}\frac{9}{5}\\ 0\\ -1\end{bmatrix}\Rightarrow\begin{bmatrix}v_1\\ v_2\\ v_{Th}\end{bmatrix}=\begin{bmatrix}8.8\\ -0.4\\ -16.4\end{bmatrix}$$

This implies that, $v_{Th}=\ -16.4V$

Let denote short-circuit current as i_{sc}*!*

Let denote the bottom node as reference node, first middle node as v_1 *and* $v_{5\Omega}$ *as* v_2*!*

By KCL for first and second node voltages gives,

$$-1+\frac{v_1-8}{10}+\frac{v_1-v_2}{10}=0$$

$$\frac{v_2-v_1}{10}+\frac{v_2}{5}+\frac{v_2}{16}=0$$

Rearranging the equations gives,

$$v_1\left(\frac{1}{10}+\frac{1}{10}\right)+v_2\left(-\frac{1}{10}\right)=\frac{9}{5}$$

$$v_1\left(-\frac{1}{10}\right)+v_2\left(\frac{1}{10}+\frac{1}{5}+\frac{1}{16}\right)=0$$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow\boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow\boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix}\frac{1}{5} & -\frac{1}{10}\\ -\frac{1}{10} & \frac{29}{80}\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}\frac{9}{5}\\ 0\end{bmatrix}\Rightarrow\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}10.44\\ 2.88\end{bmatrix}$$

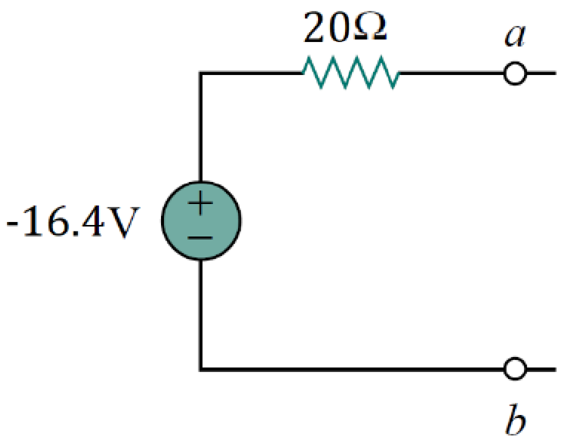
This implies that, $v_1=\ \mathbf{10.44V}$ *and* $v_2=\ \mathbf{2.88V}$

CONT...

By KCL for node a gives, $1 - \frac{v_2}{16} + i_{sc} = 0 \Rightarrow i_{sc} = \mathbf{-0.82A}$

As it known, $R_{Th} = \frac{v_{Th}}{i_5} = \mathbf{20\Omega}$

So that the Thevenin equivalent circuit is given below!



4.41

Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{ab} = v_{3\Omega}$ *directly!*

Let denote the bottom node as reference node and $v_{6\Omega}$ *as* v_1*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives,

$$-1 + \frac{v_1}{6} + \frac{v_1 + 14 - v_{Th}}{14} = 0$$

$$\frac{v_{Th} - 14 - v_1}{14} + 3 + \frac{v_{Th}}{5} = 0$$

Rearranging the equations gives,

$$v_1\left(\frac{1}{6} + \frac{1}{14}\right) + v_{Th}\left(-\frac{1}{14}\right) = 0$$

$$v_1\left(-\frac{1}{14}\right) + v_{Th}\left(\frac{1}{14} + \frac{1}{5}\right) = -2$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

CONT...

$$\begin{bmatrix} \frac{5}{21} & -\frac{1}{14} \\ -\frac{1}{14} & \frac{19}{70} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_{Th} \end{bmatrix} = \begin{bmatrix} -2.4 \\ -8 \end{bmatrix}$$

This implies that, $v_{Th} = \boldsymbol{-8V}$

Now, let create a short-circuit for the terminals a and b!

Let denote short-circuit current as i_{sc} *!*

This implies that, voltage drop across the 5Ω *resistor becomes zero!*

Let denote the bottom node as reference node and $v_{6\Omega}$ *as* v_1 *!*

By KCL for node voltages gives,

$$-1 + \frac{v_1}{6} + \frac{v_1 + 14}{14} = 0$$

$$\frac{-14 - v_1}{14} + 3 + i_{sc} = 0$$

Rearranging the equations gives,

$$v_1\left(\frac{1}{6} + \frac{1}{14}\right) + 0i_{sc} =$$

$$v_1\left(-\frac{1}{14}\right) + i_{sc} = -2$$

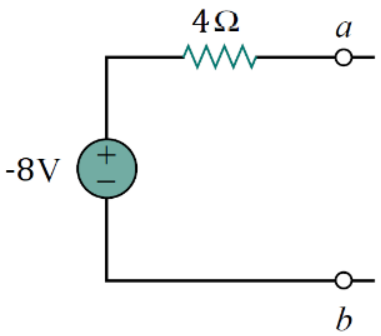
As it known, $\boldsymbol{Ax = b \Rightarrow A^{-1}Ax = A^{-1}b \Rightarrow x = A^{-1}b}$

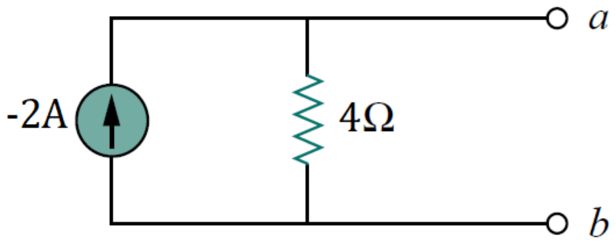
$$\begin{bmatrix} \frac{5}{21} & 0 \\ -\frac{1}{14} & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ i_{sc} \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ i_{sc} \end{bmatrix} = \begin{bmatrix} 4.2 \\ -2 \end{bmatrix}$$

This implies that, $i_{sc} = \boldsymbol{-2A}$

As it known, $I_N = i_{sc} = \boldsymbol{-2A}$ *and* $R_N = R_{Th} = \frac{v_{Th}}{i_{sc}} = \boldsymbol{4\Omega}$

So that the Thevenenin and Norton equaivalent circuits are given below!





4.43

The load resistor will be ignored in order to find the open-circuit voltage!

Because of we are trying to find Thevenin equivalent, v_{Th} *is equal to open-circuit voltage directly!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections of resistors!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

It's obvious that, 10Ω *and* 10Ω *are connected in series so* R_{eq} *for them is* $10\Omega + 10\Omega = 20\Omega$

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

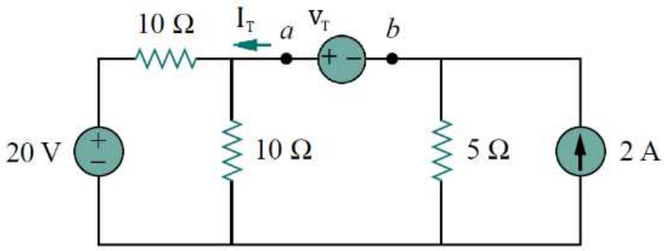
This implies, $v_a = \frac{10}{20} 20V = 10V$

It's clear that, 5Ω *and* $2A$ *source are connected in same pair of terminals!*

This implies that, $v_b = 2A \cdot 5\Omega = 10V$

So that, $v_{Th} = v_{ab} = v_a - v_b = 10 - 10 = 0$

Now in order to find the R_{Th}*, let redraw the circuit with test a test source!*



As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

Let denote the bottom node as reference node and $v_{10\Omega}$ *as* v_1 *and* $v_{5\Omega}$ *as* v_2*!*

It's obvious that independent voltage source V_T *connects two non-reference nodes!*

This implies that, V_T *source creates a supernode!*

By KCL for supernode, $\frac{v_1 - 20}{10} + \frac{v_1}{10} + \frac{v_2}{5} - 2 = 0$

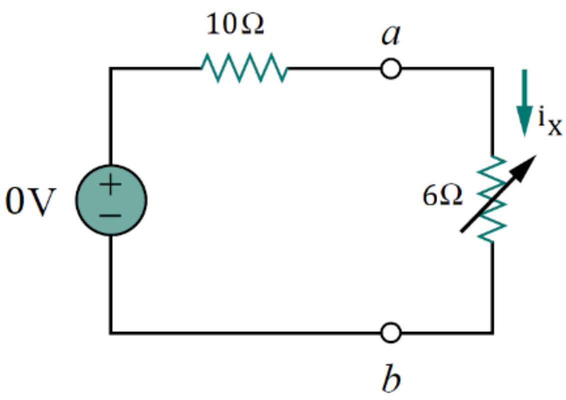
It's clear that, $v_T = v_1 - v_2$ *and* $I_T = \frac{v_1}{10} + \frac{v_1 - 20}{10}$

By using substitution, $10I_T = V_T$

As it known, $R_{Th} = \frac{V_T}{I_T} = \mathbf{10\Omega}$

CONT...

So that the Thevenin equivalent circuit becomes,



4.45

Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{ab} = v_{4\Omega}$ *directly!*

Let denote the bottom node as reference node and $v_{6\Omega}$ *as* v_1*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives,

$$-6 + \frac{v_1}{6} + \frac{v_1 - v_{Th}}{6} = 0$$

$$\frac{v_{Th} - v_1}{6} + \frac{v_{Th}}{4} = 0$$

Rearranging the equations gives,

$$v_1\left(\frac{1}{6} + \frac{1}{6}\right) + v_{Th}\left(-\frac{1}{6}\right) = 6$$

$$v_1\left(-\frac{1}{6}\right) + v_{Th}\left(\frac{1}{6} + \frac{1}{4}\right) = 0$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} \frac{1}{3} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{5}{12} \end{bmatrix}\begin{bmatrix} v_1 \\ v_{Th} \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_{Th} \end{bmatrix} = \begin{bmatrix} 22.5 \\ 9 \end{bmatrix}$$

This implies that, $v_{Th} = 9V$

Now, let create a short-circuit for the terminals a and b!

CONT...

Let denote short-circuit current as i_{sc}*!*

This implies that, voltage drop across the 4Ω *resistor becomes zero!*

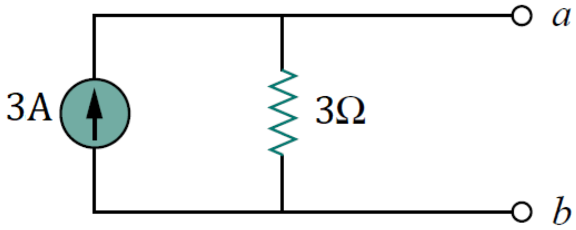
Let denote the bottom node as reference node and $v_{6\Omega}$ *as* v_1*!*

By KCL for node voltage v_1 *gives,* $-6 + \frac{v_1}{6} + \frac{v_1}{6} = 0 \Rightarrow v_1 = 18V$

It's clear that, $i_{sc} = \frac{v_1}{6} = 3A$

As it known, $I_N = i_{sc} = \boldsymbol{3A}$ *and* $R_N = R_{Th} = \frac{v_{Th}}{i_{sc}} = \boldsymbol{3\Omega}$

So that the Norton equivalent circuit becomes,



4.47

Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{ab} = V_x$ *directly!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltage gives, $\frac{v_{Th}-50}{12} + \frac{v_{Th}}{60} + 2v_{Th} = 0 \Rightarrow v_{Th} = 1.9841V$

Now, let create a short-circuit for the terminals a and b!

Let denote short-circuit current as i_{sc}*!*

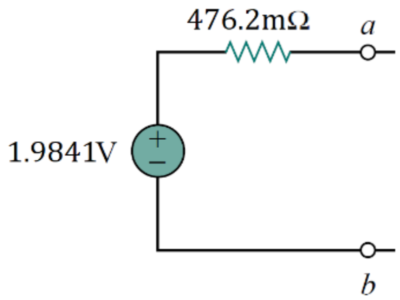
This implies that, voltage drop across the 60Ω *resistor becomes zero therefore dependent current source becomes zero!*

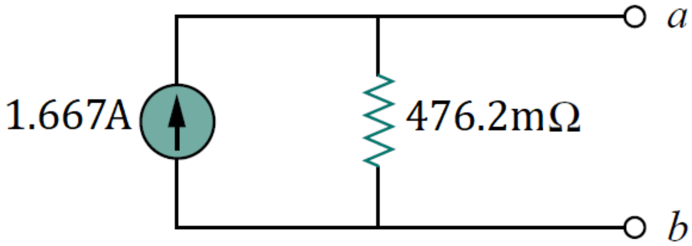
By KCL for node voltage gives, $\frac{-50}{12} + i_{sc} = 0 \Rightarrow i_{sc} = 4.1667A$

As it known, $I_N = i_{sc} = \boldsymbol{4.1667A}$ *and* $R_N = R_{Th} = \frac{v_{Th}}{i_{sc}} = 0.47620\Omega = \boldsymbol{476.2m\Omega}$

So that the Thevenin and Norton equivalent circuit are given below!

CONT...





4.51

(a) Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{ab} = v_{4\Omega}$ *directly!*

Let denote the bottom node as reference node, $v_{3\Omega}$ *as* v_1 *and* $v_{2\Omega}$ *as* v_2*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives,

$$\frac{v_1 - 120}{6} + \frac{v_1}{3} + \frac{v_1 - v_2}{4} = 0$$

$$\frac{v_2 - v_1}{4} - 6 + \frac{v_2}{2} = 0$$

Rearranging the equations gives,

$$v_1\left(\frac{1}{6} + \frac{1}{3} + \frac{1}{4}\right) + v_2\left(-\frac{1}{4}\right) = 20$$

$$v_1\left(-\frac{1}{4}\right) + v_2\left(\frac{1}{4} + \frac{1}{2}\right) = 6$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{4} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 33 \\ 19 \end{bmatrix}$$

This implies that, $v_1 = 33V$ *and* $v_2 = 19V$

It's clear that, $v_{Th} = v_{ab} = v_1 - v_2 = 14V$

Now, let create a short-circuit for the terminals a and b!

Let denote short-circuit current as i_{sc}*!*

This implies that, voltage drop across the 4Ω *resistor becomes zero!*

Let denote the bottom node as reference node, $v_{3\Omega}$ *as* v_1 *and* $v_{2\Omega}$ *as* v_2*!*

By KCL for node voltages gives,

CONT...

$\frac{v_1-120}{6}+\frac{v_1}{3}+i_{sc}=0$

$-i_s-6+\frac{v_2}{2}=0$

It's obvious that $v_{4\Omega}=v_1-v_2=0 \Rightarrow v_1=v_2$

Rearranging the equations gives,

$v_1\left(\frac{1}{6}+\frac{1}{3}\right)+i_{sc}=20$

$v_1\left(\frac{1}{2}\right)-i_{sc}=6$

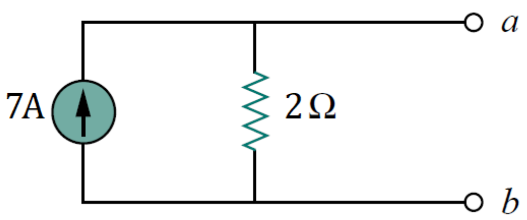
As it known, $\boldsymbol{Ax=b \Rightarrow A^{-1}Ax=A^{-1}b \Rightarrow x=A^{-1}b}$

$\begin{bmatrix}\frac{1}{2} & 1\\ \frac{1}{2} & -1\end{bmatrix}\begin{bmatrix}v_1\\ i_{sc}\end{bmatrix}=\begin{bmatrix}20\\ 6\end{bmatrix}\Rightarrow\begin{bmatrix}v_1\\ i_{sc}\end{bmatrix}=\begin{bmatrix}26\\ 7\end{bmatrix}$

This implies that, $i_{sc}=\boldsymbol{7A}$

As it known, $I_N=i_{sc}=\boldsymbol{7A}$ *and* $R_N=R_{Th}=\frac{v_{Th}}{i_{sc}}=\boldsymbol{2\Omega}$

So that the Norton equivalent circuit is given below!



(b) Because of we are trying to find Thevenin equivalent, $v_{Th}=v_{cd}=v_{2\Omega}$ *directly!*

Let denote the bottom node as reference node, $v_{3\Omega}$ *as* v_1*!*

By Ohm's Law, $v=i\cdot R \Rightarrow i=\frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i=\frac{v_{higher}-v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives,

$\frac{v_1-120}{6}+\frac{v_1}{3}+\frac{v_1-v_{Th}}{4}=0$

$\frac{v_{Th}-v_1}{4}-6+\frac{v_{Th}}{2}=0$

CONT...

Rearranging the equations gives,

$v_1\left(\frac{1}{6}+\frac{1}{3}+\frac{1}{4}\right)+v_{Th}\left(-\frac{1}{4}\right)=20$

$v_1\left(-\frac{1}{4}\right)+v_{Th}\left(\frac{1}{4}+\frac{1}{2}\right)=6$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow\boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow\boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix}\frac{3}{4} & -\frac{1}{4}\\ -\frac{1}{4} & \frac{3}{4}\end{bmatrix}\begin{bmatrix}v_1\\ v_{Th}\end{bmatrix}=\begin{bmatrix}20\\ 6\end{bmatrix}\Rightarrow\begin{bmatrix}v_1\\ v_{Th}\end{bmatrix}=\begin{bmatrix}33\\ 19\end{bmatrix}$$

This implies that, $v_{Th}=19V$

Now, let create a short-circuit for the terminals c and d!

Let denote short-circuit current as i_{sc}*!*

This implies that, voltage drop across the 2Ω *resistor becomes zero!*

Let denote the bottom node as reference node, $v_{3\Omega}$ *as* v_1*!*

By KCL for node voltage v_1 *gives,* $\frac{v_1-120}{6}+\frac{v_1}{3}+\frac{v_1}{4}=0$

By KCL for node b gives, $-\frac{v_1}{4}-6+i_{sc}=0$

Rearranging the equations gives,

$v_1\left(\frac{1}{6}+\frac{1}{3}+\frac{1}{4}\right)+0i_{sc}=20$

$v_1\left(-\frac{1}{4}\right)+i_{sc}=6$

As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow\boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow\boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix}\frac{3}{4} & 0\\ -\frac{1}{4} & 1\end{bmatrix}\begin{bmatrix}v_1\\ i_{sc}\end{bmatrix}=\begin{bmatrix}20\\ 6\end{bmatrix}\Rightarrow\begin{bmatrix}v_1\\ i_{sc}\end{bmatrix}=\begin{bmatrix}26.667\\ 12.667\end{bmatrix}$$

This implies that, $i_{sc}=\mathbf{12.667A}$

As it known, $I_N=i_{sc}=\mathbf{12.667A}$ *and* $R_N=R_{Th}=\frac{v_{Th}}{i_{sc}}=\mathbf{1.5\Omega}$

4.53

Because of we are trying to find Thevenin equivalent, $v_{Th}=v_{ab}$ *directly!*

Let denote the bottom node as reference node!

By Ohm's Law, $v=i\cdot R\Rightarrow i=\frac{v}{R}$

As it known, current flows from a higher potential to lower!

CONT...

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives,

$\frac{v_0 - 18}{6} - 0.25v_0 + \frac{v_0}{3} + \frac{v_0 - v_{Th}}{2} = 0$

$0.25v_0 + \frac{v_{Th} - v_0}{2} = 0$

Rearranging the equations gives,

$v_0\left(\frac{1}{6} + \frac{1}{3} + \frac{1}{2} - \frac{1}{4}\right) + v_{Th}\left(-\frac{1}{2}\right) = 3$

$v_0\left(\frac{1}{4} - \frac{1}{2}\right) + v_{Th}\left(\frac{1}{2}\right) = 0$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} \frac{3}{4} & -\frac{1}{2} \\ -\frac{1}{4} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} v_0 \\ v_{Th} \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_0 \\ v_{Th} \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

This implies that, $v_{Th} = 3V$

Now, let create a short-circuit for the terminals c and d!

Let denote short-circuit current as i_{sc}*!*

By KCL for node voltage v_0 *gives,* $\frac{v_0 - 18}{6} - 0.25v_0 + \frac{v_0}{3} + \frac{v_0}{2} = 0$

It's clear that, $0.25v_0 - \frac{v_0}{2} + i_{sc} = 0$

Rearranging the equations gives,

$v_0\left(\frac{1}{6} + \frac{1}{3} + \frac{1}{2} - \frac{1}{4}\right) + 0i_{sc} = 3$

$v_0\left(\frac{1}{4} - \frac{1}{2}\right) + i_{sc} = 0$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

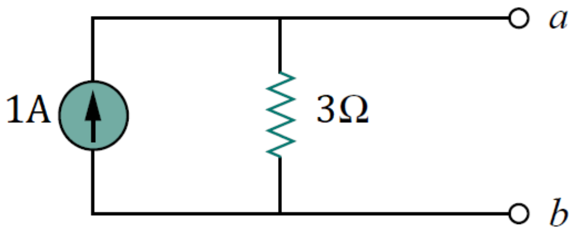
$$\begin{bmatrix} \frac{3}{4} & 0 \\ -\frac{1}{4} & 1 \end{bmatrix} \begin{bmatrix} v_0 \\ i_{sc} \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_0 \\ i_{sc} \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

This implies that, $i_{sc} = \boldsymbol{1A}$

As it known, $I_N = i_{sc} = \boldsymbol{1A}$ *and* $R_N = R_{Th} = \frac{v_{Th}}{i_{sc}} = \boldsymbol{3\Omega}$

CONT . . .

So that the Norton equivalent circuit is given below!



4.55

Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{ab} = V_{ab} = v_{50k\Omega}$ *directly!*

Let denote the bottom node as reference node!

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

Assume that direction of voltage drops signed positive!

By KVL for leftmost closed path gives,

$-2 + 8000I + 0.001V_{ab} = 0$

It's clear that, $V_{ab} = -400000I$

Rearranging the equations gives,

$8000I + 0.001V_{ab} = 2$

$4000000I + V_{ab} = 0$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 8000 & 0.001 \\ 4000000 & 1 \end{bmatrix} \begin{bmatrix} I \\ V_{ab} \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} I \\ V_{ab} \end{bmatrix} = \begin{bmatrix} 5 \times 10^{-4} \\ -2000 \end{bmatrix}$$

This implies, $V_{ab} = v_{Th} = -2000V = -2kV$

Now, let create a short-circuit for the terminals a and b!

Let denote short-circuit current as i_{sc}*!*

This implies that, voltage drop across the $50k\Omega$ *resistor becomes zero therefore dependent voltage source becomes zero!*

By KVL for leftmost closed path gives, $-2 + 8000I = 0$

It's clear that, $80I = -i_s$

Rearranging the equations gives,

$8000I + 0i_s = 2$

$80I + i_s = 0$

CONT...

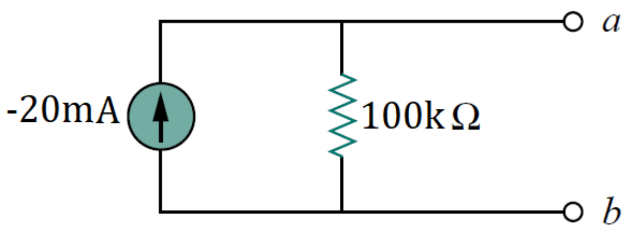
As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 8000 & 0 \\ 80 & 1 \end{bmatrix} \begin{bmatrix} I \\ i_{sc} \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} I \\ i_{sc} \end{bmatrix} = \begin{bmatrix} 2.5 \times 10^{-4} \\ -0.02 \end{bmatrix}$$

This implies, $i_{sc} = -0.02A = -20mA$

As it known, $I_N = i_{sc} = \boldsymbol{-20mA}$ *and* $R_N = R_{Th} = \frac{v_{Th}}{i_{sc}} = 100000\Omega = \boldsymbol{100k\Omega}$

So that the Norton equivalent circuit is given below!



4.57

Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{ab} = v_{10\Omega}$ *directly!*

Let denote the bottom node as reference node!

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives,

$$\frac{v_x - 50}{3} + \frac{v_x}{6} + \frac{v_x - v_{Th}}{2} = 0$$

$$\frac{v_{Th} - v_x}{2} - 0.5v_x + \frac{v_{Th}}{10} = 0$$

Rearranging the equations gives,

$$v_x\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{2}\right) + v_{Th}\left(-\frac{1}{2}\right) = \frac{50}{3}$$

$$v_x\left(-\frac{1}{2} - \frac{1}{2}\right) + v_{Th}\left(\frac{1}{2} + \frac{1}{10}\right) = 0$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 1 & -\frac{1}{2} \\ -1 & \frac{3}{5} \end{bmatrix} \begin{bmatrix} v_x \\ v_{Th} \end{bmatrix} = \begin{bmatrix} \frac{50}{3} \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_x \\ v_{Th} \end{bmatrix} = \begin{bmatrix} 100 \\ 166.67 \end{bmatrix}$$

This implies, $v_{Th} = \boldsymbol{166.67V}$

CONT...

Now, let create a short-circuit for the terminals a and b!

Let denote short-circuit current as i_{sc}*!*

This implies that, voltage drop across the 10Ω *resistor becomes zero!*

By KCL for node voltages gives,

$$\frac{v_x - 50}{3} + \frac{v_x}{6} + \frac{v_x}{2} = 0$$

$$-\frac{v_x}{2} - 0.5v_x + i_{sc} = 0$$

Rearranging the equations gives,

$$v_x\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{2}\right) + 0i_{sc} = \frac{50}{3}$$

$$v_x\left(-\frac{1}{2} - \frac{1}{2}\right) + i_{sc} = 0$$

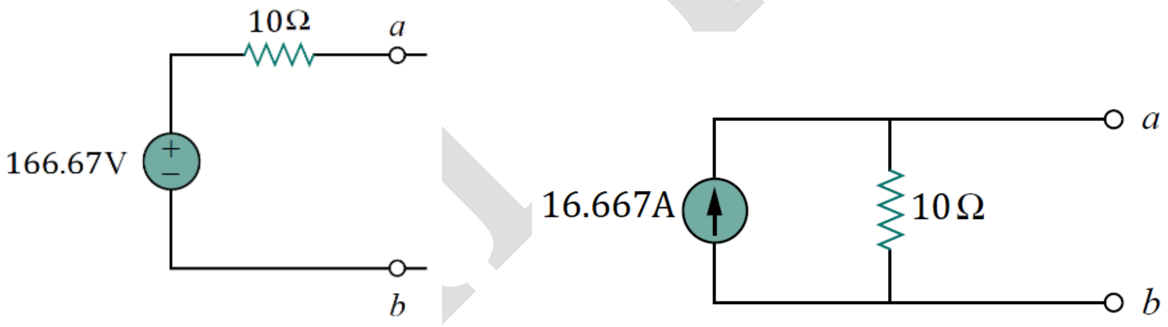
As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} v_x \\ i_{sc} \end{bmatrix} = \begin{bmatrix} \frac{50}{3} \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_x \\ i_{sc} \end{bmatrix} = \begin{bmatrix} 16.667 \\ 16.667 \end{bmatrix}$$

This implies, $i_{sc} = 16.667A$

As it known, $I_N = i_{sc} = \mathbf{16.667A}$ *and* $R_N = R_{Th} = \frac{v_{Th}}{i_{sc}} = \mathbf{10\Omega}$

So that the Thevenin and Norton equivalent circuits are given below!



4.59

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections and* $R_{eq} = \left(\sum_{n=1}^{\infty}\frac{1}{R_n}\right)^{-1}$ *for parallel connections of resistors!*

Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{ab}$ *directly!*

CONT...

By Current Division equation, $i_x = \frac{R_{eq}}{R_x} i_{source}$

It's clear that, $i_{10\Omega} = i_{50\Omega}$ *and* $i_{20\Omega} = i_{40\Omega}$*!*

It's obvious that, 10Ω *and* 50Ω *are connected in series so that* R_{eq} *for them is* $10\Omega + 50\Omega = 60\Omega$

Likewise, 20Ω *and* 40Ω *are connected in series so that* R_{eq} *for them is* $20\Omega + 40\Omega = 60\Omega$

Now, 60Ω *and* 60Ω *are connected in parallel so that* R_{eq} *for them is* $\frac{1}{60} + \frac{1}{60} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = 30\Omega$

By substitution, $i_{10\Omega} = i_{50\Omega} = \frac{30}{60} 8A = 4A$ *and* $i_{20\Omega} = i_{40\Omega} = \frac{30}{60} 8A = 4A$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

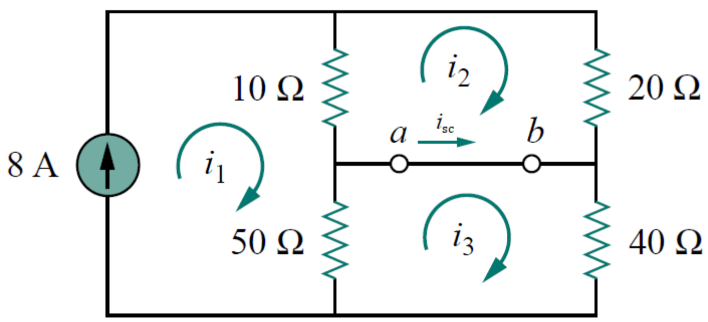
This implies, $v_a = v_{50\Omega} = 50 i_{50\Omega} = 200V$ *and* $v_b = v_{40\Omega} = 40 i_{40\Omega} = 160V$

It's clear that, $v_{Th} = v_{ab} = v_a - v_b = 40V$

Now, let create a short-circuit for the terminals a and b!

Let denote short-circuit current as i_{sc}*!*

Let redraw the circuit with mesh currents!



Assume that direction of voltage drops signed positive!

It's clear that, $i_1 = \mathbf{8A}$*!*

By KVL for other meshes gives,

$20i_2 + 10(i_2 - 8) = 0$

$40i_3 + 50(i_3 - 8) = 0$

Rearranging the equations gives,

$30i_2 + 0i_3 = 80$

$0i_2 + 90i_3 = 400$

As it known, $\boldsymbol{Ax = b \Rightarrow A^{-1}Ax = A^{-1}b \Rightarrow x = A^{-1}b}$

CONT...

$$\begin{bmatrix} 30 & 0 \\ 0 & 90 \end{bmatrix}\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 80 \\ 400 \end{bmatrix} \Rightarrow \begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 2.6667 \\ 4.4445 \end{bmatrix}$$

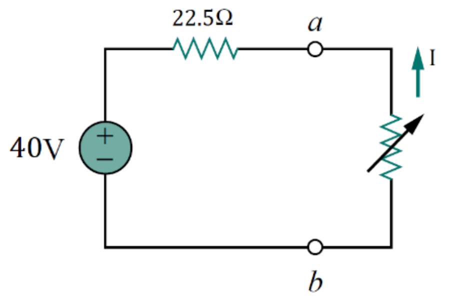
This implies, $i_2 = 2.6667A$ *and* $i_3 = 4.4445A$

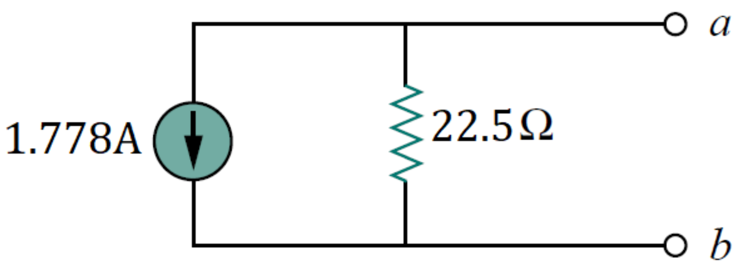
It's clear that, $i_{sc} = i_3 - i_2 = -1.7778A$

Note that, negative sign indicates that the current flows in the direction opposite to the one assumed, that is, from terminal b to a!

As it known, $I_N = i_{sc} = \mathbf{1.7778A}$ *and* $R_N = R_{Th} = \frac{v_{Th}}{i_{sc}} = \mathbf{22.5\Omega}$

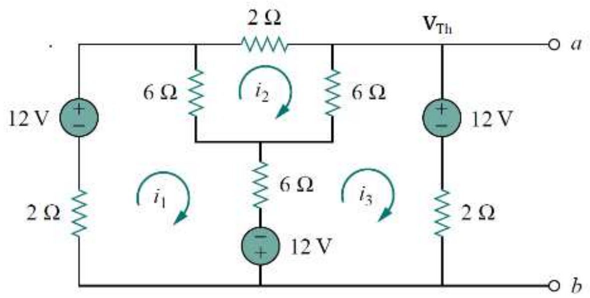
So that the Thevenin and Norton equivalent circuits are given below!





4.61

Let redraw the circuit with mesh currents!



Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{ab}$ *directly!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

Assume that direction of voltage drops signed positive!

By KVL for each mesh gives,

$2i_1 - 12 + 6(i_1 - i_2) + 6(i_1 - i_3) - 12 = 0$

$2i_2 + 6(i_2 - i_3) + 6(i_2 - i_1) = 0$

$12 + 2i_3 + 12 + 6(i_3 - i_1) + 6(i_3 - i_2) = 0$

Rearranging the equations gives

CONT. . .

$14i_1 - 6i_2 - 6i_3 = 24$

$-6i_1 + 14i_2 - 6i_3 = 0$

$-6i_1 - 6i_2 + 14i_3 = -24$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

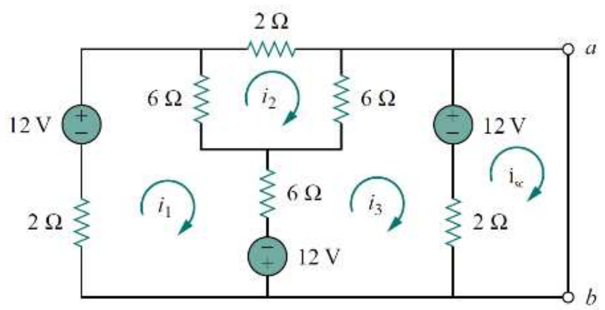
$$\begin{bmatrix} 14 & -6 & -6 \\ -6 & 14 & -6 \\ -6 & -6 & 14 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_2 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ -24 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1.2 \\ 0 \\ -1.2 \end{bmatrix}$$

This implies that, $i_1 = \ 1.2A,\ i_2 = 0A \ and \ i_3 = -1.2$

It's clear that, $v_{Th} = 12 + 2i_3 = 9.6V$

Let denote short-circuit current as i_{sc}*!*

Let redraw the circuit with mesh currents!



By KVL for each mesh gives,

$2i_1 - 12 + 6(i_1 - i_2) + 6(i_1 - i_3) - 12 = 0$

$2i_2 + 6(i_2 - i_3) + 6(i_2 - i_1) = 0$

$12 + 2(i_3 - i_{sc}) + 12 + 6(i_3 - i_1) + 6(i_3 - i_2) = 0$

$2(i_{sc} - i_3) - 12 = 0$

Rearranging the equations gives,

$14i_1 - 6i_2 - 6i_3 + 0i_{sc} = 24$

$-6i_1 + 14i_2 - 6i_3 + 0i_{sc} = 0$

$-6i_1 - 6i_2 + 14i_3 - 2i_{sc} = -24$

$0i_1 + 0i_2 - 2i_3 + 2i_{sc} = 12$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

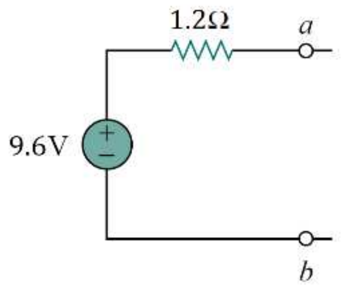
$$\begin{bmatrix} 14 & -6 & -6 & 0 \\ -6 & 14 & -6 & 0 \\ -6 & -6 & 14 & -2 \\ 0 & 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_{sc} \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ -24 \\ 12 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_{sc} \end{bmatrix} = \begin{bmatrix} 3.6 \\ 2.4 \\ 2 \\ 8 \end{bmatrix}$$

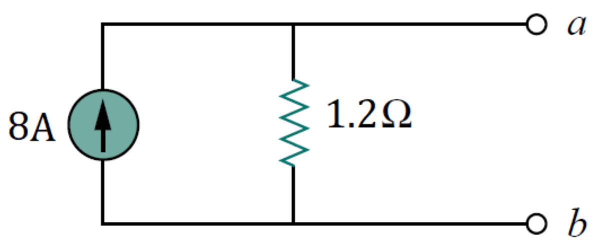
This implies that, $i_{sc} = \boldsymbol{8A}$

CONT...

As it known, $I_N = i_{sc} = \mathbf{8A}$ *and* $R_N = R_{Th} = \frac{v_{Th}}{i_{sc}} = \mathbf{1.2\Omega}$

So that the Thevenin and Norton equivalent circuits are given below!





4.63

Because of we are trying to find Thevenin equivalent, $v_{Th} = v_{0.5v_0}$ *directly!*

Let denote the bottom node as reference node!

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltage gives, $\frac{v_T - v_0}{10} - 0.5v_0 = 0$

It's clear that, $\frac{v_0}{20} = \frac{v_{Th} - v_0}{10}$

Rearranging the equations gives,

$$v_0\left(-\frac{1}{10} - \frac{1}{2}\right) + v_{Th}\left(\frac{1}{10}\right) = 0$$

$$v_0\left(\frac{1}{20} + \frac{1}{10}\right) + v_{Th}\left(-\frac{1}{10}\right) = 0$$

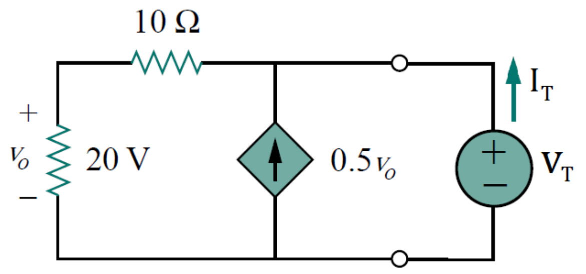
As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} -\frac{3}{5} & -\frac{1}{10} \\ \frac{3}{20} & -\frac{1}{10} \end{bmatrix}\begin{bmatrix} v_0 \\ v_{Th} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_0 \\ v_{Th} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This implies, $v_{Th} = \mathbf{0V}$

Let redraw the circuit with test voltage source and current in order to find Norton or Thevenin resistance!

CONT . . .



It's obvious that, $v_T = v_{0.5v_0}$ *because they are connected to the same pair of terminals!(if there is any confusion you can verify it using Kirchhoff's Voltage Law!)*

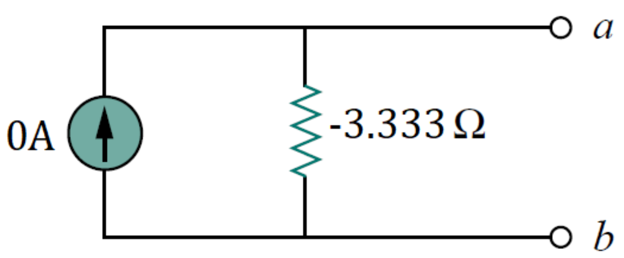
By KCL for node voltage gives, $\frac{v_T - v_0}{10} - 0.5v_0 - I_T = 0$

It's clear that, $\frac{v_0}{20} = \frac{v_T - v_0}{10}$

By using substitution, $-10I_T = 3v_T$

As it known, $R_{Th} = R_N = \frac{v_T}{I_T} = \mathbf{-3.333\Omega}$

So that the Norton equivalent circuit is given below!



4.65

The voltage v_0 *will be ignored in order to find the open-circuit voltage!*

Open-circuit causes zero voltage drop across the 2Ω *resistor!*

This implies, $v_{Th} = v_{12\Omega}$*!*

Let denote the bottom node as reference node!

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltage gives, $\frac{v_{Th} - 64}{4} + \frac{v_{Th}}{12} = 0 \Rightarrow v_{Th} = \mathbf{48V}$

CONT...

Now, let create a short-circuit for the terminals a and b!

Let denote short-circuit current as i_{sc} *and* $v_{12\Omega}$ *as* v_1*!*

By KCL for node voltage v_1 *gives,* $\frac{v_1-64}{4}+\frac{v_1}{12}+\frac{v_1}{2}=0$

It's clear that, $\frac{v_1}{2}=i_{sc}$*!*

Rearranging the equations gives,

$v_1\left(\frac{1}{4}+\frac{1}{12}+\frac{1}{2}\right)+0i_{sc}=16$

$v_1\left(\frac{1}{2}\right)-i_{sc}=0$

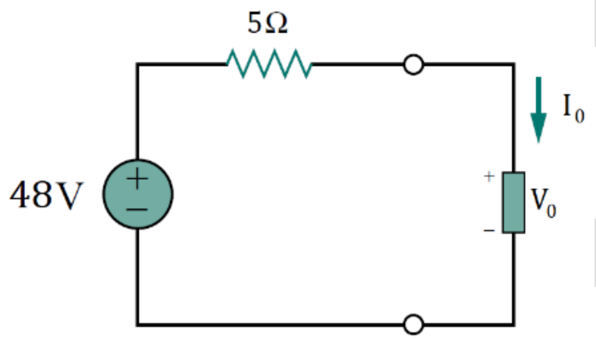
As it known, $\boldsymbol{Ax}=\boldsymbol{b}\Rightarrow\boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{b}\Rightarrow\boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix}\frac{5}{6} & 0\\ \frac{1}{2} & -1\end{bmatrix}\begin{bmatrix}v_1\\ i_{sc}\end{bmatrix}=\begin{bmatrix}16\\ 0\end{bmatrix}\Rightarrow\begin{bmatrix}v_1\\ i_{sc}\end{bmatrix}=\begin{bmatrix}19.2\\ 9.6\end{bmatrix}$$

This implies, $i_{sc}=9.6A$

As it known, $R_{Th}=R_N=\frac{v_{Th}}{i_{sc}}=\boldsymbol{5\Omega}$

Let redraw the Thevenin equivalent circuit with the circuit element now!



Assume that direction of voltage drops signed positive!

By KVL for closed path gives, $-48+5I_0+V_0=0\Rightarrow V_0=48-5I_0$

4.67

The load resistor will be ignored in order to find the open-circuit voltage!

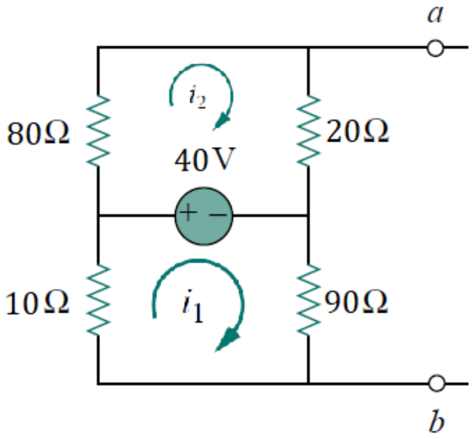
Because of we are trying to find Thevenin equivalent, v_{Th} *is equal to open-circuit voltage directly!*

By Ohm's Law, $v=i\cdot R\Rightarrow i=\frac{v}{R}$

Assume that direction of voltage drops signed positive!

CONT...

Let redraw the circuit with mesh currents!



By KVL for each mesh gives,

$90i_1 + 10i_1 + 40 = 0 \Rightarrow i_1 = -0.4A = -400mA$

$20i_2 - 40 + 80i_2 = 0 \Rightarrow i_2 = 0.4A = 400mA$

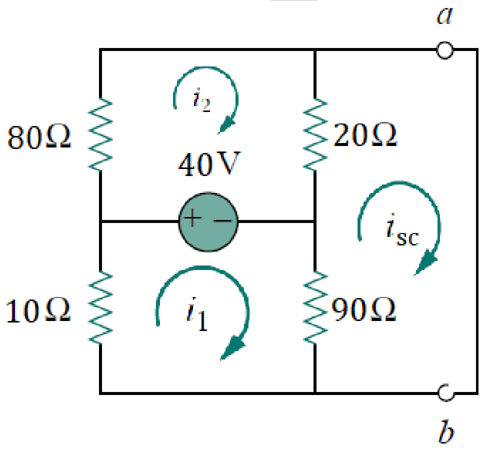
Let denote the bottom node as reference node!

This implies, $v_{Th} = v_{20\Omega} + v_{90\Omega} = 20i_1 + 90i_2 = \mathbf{-28V}$

Now, let create a short-circuit for the terminals a and b!

Let denote short-circuit current as i_{sc}*!*

Let redraw the circuit with mesh currents!



By KVL for each mesh gives,

$90(i_1 - i_{sc}) + 10i_1 + 40 = 0$

$20(i_2 - i_{sc}) - 40 + 80i_2 = 0$

$90(i_{sc} - i_1) + 20(i_{sc} - i_2) = 0$

Rearranging the equations gives,

CONT...

$100i_1 + 0i_2 - 90i_{sc} = -40$

$0i_1 + 100i_2 - 20i_{sc} = 40$

$-90i_1 - 20i_2 + 110i_{sc} = 0$

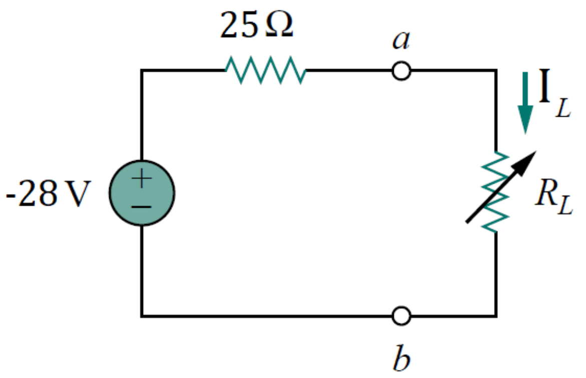
As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} 100 & 0 & -90 \\ 0 & 100 & -20 \\ -90 & -20 & 110 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_2 \end{bmatrix} = \begin{bmatrix} -40 \\ 40 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_{sc} \end{bmatrix} = \begin{bmatrix} -1.408 \\ 0.176 \\ -1.12 \end{bmatrix}$$

This implies that, $i_{sc} = -1.12A$

As it known, $R_{Th} = \frac{v_{Th}}{i_{sc}} = \boldsymbol{25\Omega}$

So that the Thevenin equivalent is given below!



Here R_L *corresponds to* R*!*

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i = i^2 \cdot R$

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections!*

This implies that, R_{eq} *for* R_{Th} *and* R_L *becomes* $R_{eq} = R_{Th} + R_L$

Now, $p = \left(\frac{v_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L$

In order to find the maximum power that transferred to load, we must differentiate the equation given above!

So that, $\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right] = V_{Th}^2 \left[\frac{R_{Th}-R_L}{(R_{Th}+R_2)^3}\right] = \boldsymbol{0}$

This implies that, the maximum power is transferred to the load when the load resistance equals to the Thevenin resistance!($R_L = R_{Th}$*!)*

So that, $p_{max} = \left(\frac{v_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L = \left(\frac{v_{Th}}{2R_{Th}}\right)^2 \cdot R_{Th} = \frac{v_{Th}^2}{4R_{Th}} = \boldsymbol{7.84W}$

4.69

The load resistor will be ignored in order to find the open-circuit voltage!

Because of we are trying to find Thevenin equivalent, v_{Th} *is equal to open-circuit voltage and voltage drop across the* $30k\Omega$ *resistor directly!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltages gives,

$$\frac{v_0 - 100}{10000} + \frac{v_0}{40000} + \frac{v_0 - v_{Th}}{22000} = 0$$

$$\frac{v_{Th} - v_0}{22000} - 0.003v_0 + \frac{v_{Th}}{30000} = 0$$

Rearranging the equations gives,

$$v_0\left(\frac{1}{10} + \frac{1}{40} + \frac{1}{22}\right) + v_{Th}\left(-\frac{1}{22}\right) = 10$$

$$v_0\left(-\frac{1}{22} - 3\right) + v_{Th}\left(\frac{1}{22} + \frac{1}{30}\right) = 0$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} \frac{15}{88} & -\frac{1}{22} \\ -\frac{67}{22} & \frac{13}{165} \end{bmatrix} \begin{bmatrix} v_0 \\ v_{Th} \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_0 \\ v_{Th} \end{bmatrix} = \begin{bmatrix} 6.3030 \\ -243.64 \end{bmatrix}$$

This implies, $v_{Th} = -243.64V$

Now, let create a short-circuit for the terminals a and b!

Let denote short-circuit current as i_{sc}*!*

This implies that, voltage drop across the $30k\Omega$ *resistor becomes zero!*

By KCL for node voltages gives,

$$\frac{v_0 - 100}{10000} + \frac{v_0}{40000} + \frac{v_0}{22000} = 0$$

$$\frac{-v_0}{22000} - 0.003v_0 + i_{sc} = 0$$

Rearranging the equations gives,

$$v_0\left(\frac{1}{10} + \frac{1}{40} + \frac{1}{22}\right) + 0i_{sc} = 10$$

$$v_0\left(-\frac{1}{22} - 3\right) + 1000i_{sc} = 0$$

CONT . . .

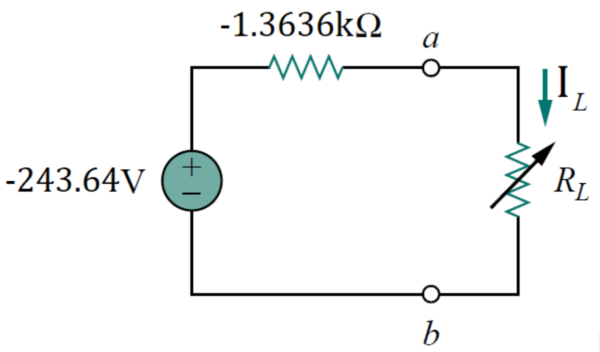
As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} \frac{15}{88} & 0 \\ -\frac{67}{22} & 1000 \end{bmatrix}\begin{bmatrix} v_0 \\ i_{sc} \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_0 \\ i_{sc} \end{bmatrix} = \begin{bmatrix} 58.667 \\ 0.17867 \end{bmatrix}$$

This implies, $i_{sc} = 0.17867A = 178.67mA$

As it known, $R_{Th} = \frac{v_{Th}}{i_{sc}} = -1363.6\Omega = -\boldsymbol{1.3636k\Omega}$

So that the Thevenin equivalent is given below!



Here R_L *corresponds to* R*!*

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i = i^2 \cdot R$

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections!*

This implies that, R_{eq} *for* R_{Th} *and* R_L *becomes* $R_{eq} = R_{Th} + R_L$

Now, $p = \left(\frac{v_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L$

In order to find the maximum power that transferred to load, we must differentiate the equation given above!

So that, $\frac{dp}{dR_L} = V_{Th}^2\left[\frac{(R_{Th}+R_L)^2-2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right] = V_{Th}^2\left[\frac{R_{Th}-R_L}{(R_{Th}+R_2)^3}\right] = \boldsymbol{0}$

This implies that, the maximum power is transferred to the load when the load resistance equals to the Thevenin resistance!($R_L = R_{Th}$*!)*

However, we are facing a special situation right now!

If we let $R_L = -R_{Th}$, *the denominator will be zero!*

So that,

$$p_{max} = \left(\frac{v_{Th}}{R_{Th} + R_L}\right)^2 \cdot R_L = \lim_{R_T+R_L\to 0} = \left(\frac{v_{Th}}{R_{Th} + R_L}\right)^2 \cdot R_L = \left(\frac{-243.64}{0}\right)^2 \cdot (1363.6\Omega) = \infty$$

So that the maximum power becomes infinity theoretically!

4.71

Because of we are trying to find Thevenin equivalent, v_{Th} *is equal to open-circuit voltage and voltage drop across the* $30k\Omega$ *resistor directly!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltage v_{Th} *gives,* $\frac{v_{Th} + 120v_0}{10000} + \frac{v_{Th}}{40000} = 0$

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections of resistors!*

It's obvious that, $3k\Omega$ *and* $1k\Omega$ *are connected in series so* R_{eq} *for them is* $3k\Omega + 1k\Omega = 4k\Omega$

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

By substitution, $v_0 = \frac{1}{4} 8V = 2V$

Rearranging the equations gives,

$v_{Th}\left(\frac{1}{10} + \frac{1}{40}\right) + 12v_0 = 0$

$0v_T + v_0 = 2$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$$\begin{bmatrix} \frac{1}{8} & 12 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{Th} \\ v_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} v_{Th} \\ v_0 \end{bmatrix} = \begin{bmatrix} -192 \\ 2 \end{bmatrix}$$

This implies, $v_{Th} = -192V$

Let denote short-circuit current as i_{sc}*!*

This implies that, voltage drop across the $40k\Omega$ *resistor becomes zero!*

By KCL for rightmost node gives, $\frac{120v_0}{10000} + i_{sc} = 0$

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

By substitution, $v_0 = \frac{1}{4} 8V = 2V$

Rearranging the equations gives,

CONT...

$v_0\left(\frac{12}{1000}\right) + i_{sc} = 0$

$v_0 + 0i_{sc} = 2$

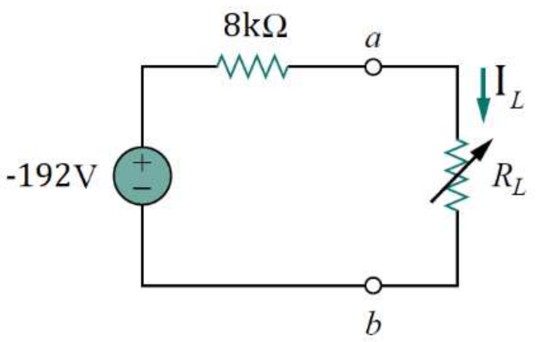
As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

$\begin{bmatrix} \frac{3}{250} & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} v_0 \\ i_{sc} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} v_0 \\ i_{sc} \end{bmatrix} = \begin{bmatrix} 2 \\ -0.024 \end{bmatrix}$

This implies, $i_{sc} = -0.024A = -24mA$

As it known, $R_{Th} = \frac{v_{Th}}{i_{sc}} = 8000\Omega = 8k\Omega$

So that the Thevenin equivalent is given below!



Here R_L *corresponds to load resistor!*

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i = i^2 \cdot R$

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections!*

This implies that, R_{eq} *for* R_{Th} *and* R_L *becomes* $R_{eq} = R_{Th} + R_L$

Now, $p = \left(\frac{v_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L$

In order to find the maximum power that transferred to load, we must differentiate the equation given above!

So that, $\frac{dp}{dR_L} = V_{Th}^2\left[\frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right] = V_{Th}^2\left[\frac{R_{Th}-R_L}{(R_{Th}+R_2)^3}\right] = 0$

This implies that, the maximum power is transferred to the load when the load resistance equals to the Thevenin resistance!($R_L = R_{Th}$ *!)*

So that, $p_{max} = \left(\frac{v_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L = \left(\frac{v_{Th}}{2R_{Th}}\right)^2 \cdot R_{Th} = \frac{v_{Th}^2}{4R_{Th}} = \mathbf{1.152W}$

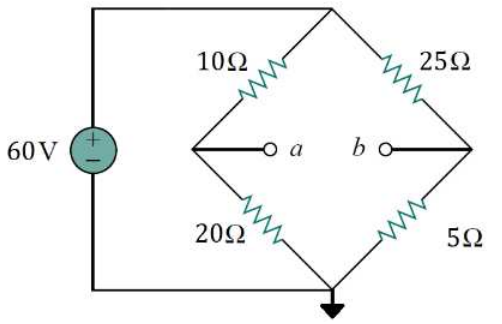
4.73

The load resistor will be ignored in order to find the open-circuit voltage!

Because of we are trying to find Thevenin equivalent, v_{Th} *is equal to open-circuit voltage directly!*

CONT...

Let redraw the circuit as open-circuit with a reference node!



By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections of resistors!*

It's obvious that, 10Ω *and* 20Ω *are connected in series so* R_{eq} *for them is* $10\Omega + 20\Omega = 30\Omega$

Likewise, 25Ω *and* 5Ω *are connected in series so* R_{eq} *for them is* $25\Omega + 5\Omega = 30\Omega$

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

It's obvious that, voltage drop across the 30Ω *resistors are same because they are both connected to the same pair of terminals!(if there is any confusion you can verify it using Kirchhoff's Voltage Law!)*

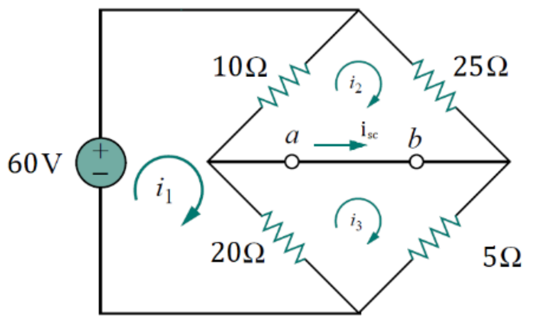
This implies, $v_{20\Omega} = v_a = \frac{20}{30} 60V = 40V$ *and* $v_{5\Omega} = v_b = \frac{5}{30} 60V = 10V$

It's obvious that, $v_{Th} = v_a - v_b = 30V$

Now, let create a short-circuit for the terminals a and b!

Let denote short-circuit current as i_{sc}*!*

Let redraw the circuit as short-circuit with mesh currents!



Assume that direction of voltage drops signed positive!

By KVL for each mesh gives,

CONT...

$-60 + 10(i_1 - i_2) + 20(i_1 - i_3) = 0$

$25i_2 + 10(i_2 - i_1) = 0$

$5i_3 + 20(i_3 - i_1) = 0$

Rearranging the equations gives,

$30i_1 - 10i_2 - 20i_3 = 60$

$-10i_1 + 35i_2 + 0i_3 = 0$

$-20i_1 + 0i_2 + 25i_3 = 0$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

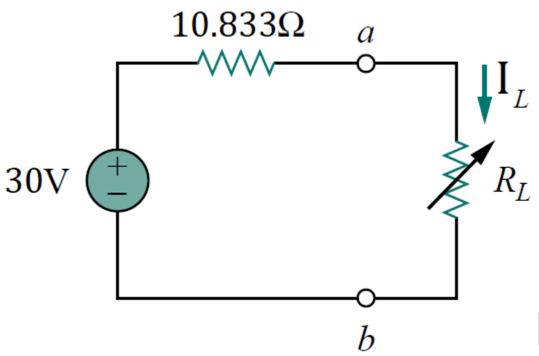
$$\begin{bmatrix} 30 & -10 & -20 \\ -10 & 35 & 0 \\ -20 & 0 & 25 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 5.3846 \\ 1.5385 \\ 4.3077 \end{bmatrix}$$

This implies that, $i_1 = 5.3846A,\ i_2 = 1.5385A$ *and* $i_3 = 4.3077A$

It's clear that, $i_s = i_3 - i_2 = 2.7692A$

As it known, $R_{Th} = \frac{v_{Th}}{i_{sc}} = \boldsymbol{10.833\Omega}$

So that the Thevenin equivalent is given below!



Here R_L *corresponds to load resistor!*

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i = i^2 \cdot R$

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections!*

This implies that, R_{eq} *for* R_{Th} *and* R_L *becomes* $R_{eq} = R_{Th} + R_L$

Now, $p = \left(\frac{v_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L$

In order to find the maximum power that transferred to load, we must differentiate the equation given above!

CONT...

So that, $\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right] = V_{Th}^2 \left[\frac{R_{Th}-R_L}{(R_{Th}+R_2)^3}\right] = \mathbf{0}$

This implies that, the maximum power is transferred to the load when the load resistance equals to the Thevenin resistance!$(R_L = R_{Th}!)$

So that, $p_{max} = \left(\frac{v_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L = \left(\frac{v_{Th}}{2R_{Th}}\right)^2 \cdot R_{Th} = \frac{v_{Th}^2}{4R_{Th}} = \mathbf{20.77W}$

4.75

The load resistor will be ignored in order to find the open-circuit voltage!

Because of we are trying to find Thevenin equivalent, v_{Th} *is equal to open-circuit voltage directly!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

Assume that direction of voltage drops signed positive!

It's clear that, $-10 - 20I + v_{Th} = 0 \Rightarrow v_{Th} = 20I + 10$

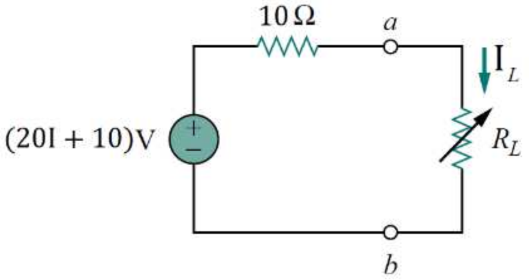
Now, let create a short-circuit for the terminals a and b!

Let denote short-circuit current as i_{sc}*!*

By KVL for closed path gives, $-10 + 10i_{sc} - 20I = 0 \Rightarrow i_{sc} = \frac{20I+10}{10}$

As it known, $R_{Th} = \frac{v_{Th}}{i_{sc}} = \mathbf{10\Omega}$

So that the Thevenin equivalent is given below!



Here R_L *corresponds to load resistor!*

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i = i^2 \cdot R$

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections!*

This implies that, R_{eq} *for* R_{Th} *and* R_L *becomes* $R_{eq} = R_{Th} + R_L$

Now, $p = \left(\frac{v_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L$

In order to find the maximum power that transferred to load, we must differentiate the equation given above!

CONT...

So that, $\frac{dp}{dR_L} = v_{Th}^2\left[\frac{(R_{Th}+R_L)^2-2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right] = v_{Th}^2\left[\frac{R_{Th}-R_L}{(R_{Th}+R_2)^3}\right] = 0$

This implies that, the maximum power is transferred to the load when the load resistance equals to the Thevenin resistance!($R_L = R_{Th}$*!)*

So that, $p_{max} = \left(\frac{v_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L = \left(\frac{v_{Th}}{2R_{Th}}\right)^2 \cdot R_{Th} = \frac{v_{Th}^2}{4R_{Th}} = \frac{(20I+10)^2}{40}$

This implies, maximum power is transferred as I approaches to infinity!

So that, $\lim_{I\to\infty}\frac{(20I+10)^2}{40} = \infty$

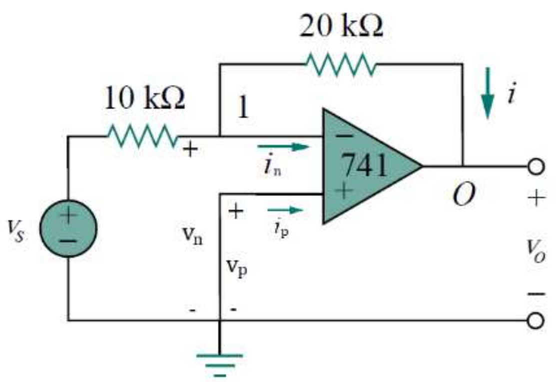
END!

Chapter 5-Practice Problems

5.2

Assume that the op amp is operating in its linear region!

Let redraw the op amp circuit with some important voltages and currents!



By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher}-v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltage v_n *gives,* $\frac{v_n-v_s}{10000} + \frac{v_n-v_0}{20000} + i_n = 0$

Rearranging the equation gives, $-2v_s - v_0 = 0!$

So that the open loop gain becomes, $\frac{v_0}{v_s} = -2$

CONT...

It's clear that, $i = \frac{v_n - v_0}{20000} = -\frac{v_0}{20000}$

When v_s *equals* $2V$, v_0 *becomes* $-4V$!

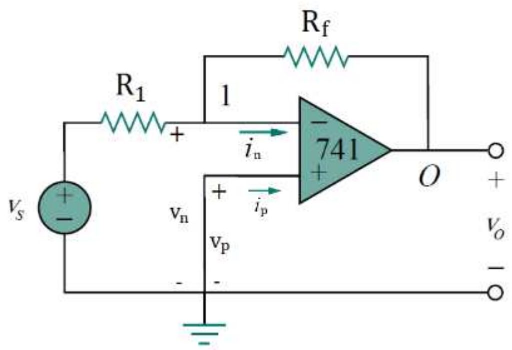
This implies, $i = -\frac{v_0}{20000} = 2 \times 10^{-4}A = \mathbf{0.2mA}$

5.3

Assume that the op amp is operating in its linear region!

Let denote, $30mV$ *as* v_s, $3k\Omega$ *as* R_1 *and* $3k\Omega$ *as* R_f!

Now let redraw the op amp circuit with some important voltages and currents!



By the definition of ideal op amp, $i_n = i_p = 0$!

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0$!

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltage v_n *gives,* $\frac{v_n - v_s}{R_1} + \frac{v_n - v_0}{R_f} + i_n = 0$

Rearranging the equation gives, $v_0 = -\frac{R_f}{R_1} v_s$!

This implies, $v_0 = -\frac{120}{3}(30mV) = -120mV = -1.2V$

Let denote the current passing through the feedback resistor as i_f!

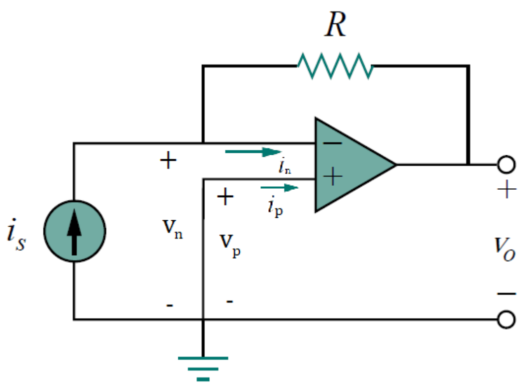
It's obvious that, $i_f = \frac{v_n - v_0}{120000} = -\frac{v_0}{120000} = 1 \times 10^{-5}A = \mathbf{10\mu A}$

5.4

(a) Assume that the op amp is operating in its linear region!

Now let redraw the op amp circuit with some important voltages and currents!

CONT...



By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

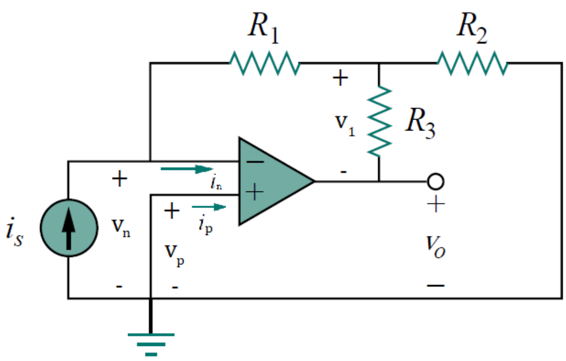
Let denote currents leaving a node positive!

By KCL for node voltage v_n *gives,* $-i_s + \frac{v_n - v_0}{R} + i_n = \mathbf{0}$

Rearranging the equation gives, $i_s = -\frac{v_0}{R} \Rightarrow -\frac{v_0}{i_s} = R \Rightarrow \frac{v_0}{i_s} = -\boldsymbol{R}$

(b) Assume that the op amp is operating in its linear region!

Now let redraw the op amp circuit with some important voltages and currents!



By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

CONT...

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltage v_n *gives,* $-i_s + \frac{v_n - v_1}{R_1} + i_n = 0$

By KCL for node v_1 *gives,* $\frac{v_1 - v_n}{R_1} + \frac{v_1 - v_0}{R_3} + \frac{v_1}{R_2} = 0$

Rearranging the equations gives,

$i_s = -\frac{v_1}{R}$

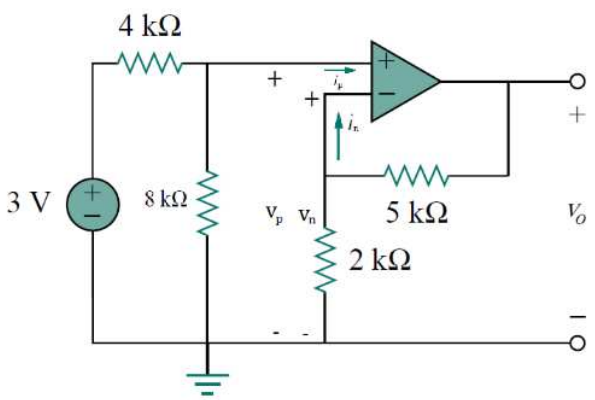
$v_0 = v_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \cdot R_3$

This implies, $\boldsymbol{\frac{v_0}{i_s} = -R_1\left(\frac{R_3}{R_1} + \frac{R_3}{R_2} + 1\right)}$

5.5

Assume that the op amp is operating in its linear region!

Now let redraw the op amp circuit with some important voltages and currents!



By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

Let denote equivalent resistance as $R_{eq}!$

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections of resistors!*

It's obvious that, 4Ω *and* 6Ω *are connected in series so* R_{eq} *for them is* $4k\Omega + 8k\Omega = 12k\Omega$

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

CONT...

By substitution, $v_p = \frac{8}{12} 3V = 2V$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node voltage $v_{2k\Omega}$ *gives,* $\frac{v_n}{2000} + i_n + \frac{v_n - v_0}{5000} = 0$

As it known, $v_n = v_p = 2V$

Rearranging the equations gives,

$v_n\left(\frac{1}{2} + \frac{1}{5}\right) + v_0\left(-\frac{1}{5}\right) = 0$

$v_n + 0v_0 = 2$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

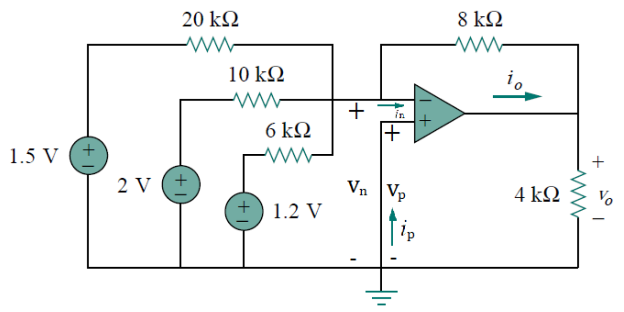
$$\begin{bmatrix} \frac{7}{10} & -\frac{1}{5} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_n \\ v_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} v_n \\ v_0 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$$

This implies, $v_0 = \boldsymbol{7V}$

5.6

Assume that the op amp is operating in its linear region!

Now let redraw the op amp circuit with some important voltages and currents!



By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

CONT...

By KCL for node v_n *gives,* $\frac{v_n-1.5}{20000}+\frac{v_n-2}{10000}+\frac{v_n-1.2}{6000}+\frac{v_n-v_0}{8000}+i_n=\mathbf{0}$

Rearranging the equation gives,

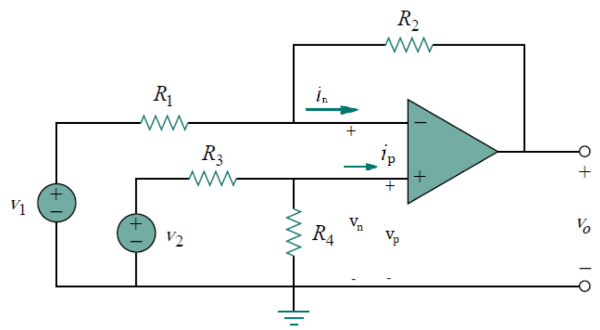
$-\frac{1.5}{20}-\frac{2}{10}-\frac{1.2}{6}-\frac{v_0}{8}=0 \Rightarrow v_0=\mathbf{-3.8V}$

By KCL for node v_0 *gives,* $\frac{v_0-v_1}{8000}-i_0+\frac{v_0}{4000} \Rightarrow i_0=\frac{3v_0}{8000}=-1.425\times10^{-3}A=\mathbf{-1.415mA}$

5.7

Assume that the op amp is operating in its linear region!

Now draw a basic difference op amp circuit with some important voltages and currents!



By the definition of ideal op amp, $i_n=i_p=0$*!*

It's obvious that, $v_p-v_n=0 \Rightarrow v_p=v_n$*!*

By Ohm's Law, $v=i\cdot R \Rightarrow i=\frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i=\frac{v_{higher}-v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_n *gives,* $\frac{v_n-v_1}{R_1}+\frac{v_n-v_0}{R_2}+i_n=0$

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq}=\sum_{n=1}^{\infty}R_n$ *for the series connections of resistors!*

It's obvious that, R_3 *and* R_4 *are connected in series so* R_{eq} *for them is* R_3+R_4*!*

By Voltage Division equation, $v_x=\frac{R_x}{R_{eq}}v_{source}$

By substitution, $v_p=\frac{R_4}{R_3+R_4}v_2$

Substituting the equations gives, $v_0=\frac{R_2(1+R_1/R_2)}{R_1(1+R_3/R_4)}v_2-\frac{R_2}{R_1}v_1$

CONT...

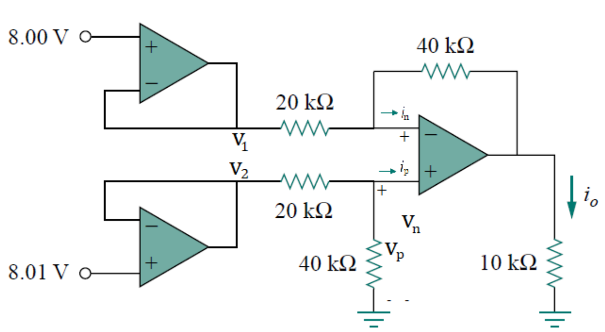
Since a difference amplifier must reject a signal common to the two inputs, the amplifier must have the property that $v_0 = 0$ *when* $v_1 = v_2$*!*

This implies, $\frac{R_1}{R_2} = \frac{R_3}{R_4}$*!*

Now by using APENDIX resistors can be, $R_1 = R_3 = \mathbf{10k\Omega}$ *and* $R_2 = R_4 = \mathbf{50k\Omega}$

5.8

Let redraw the instrumentation amplifier with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0$*!*

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n$*!*

It's clear that, $v_1 = 8.00V$ *and* $v_2 = 8.01V$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_n *gives,* $\frac{v_n - v_1}{20000} + \frac{v_n - v_{output}}{40000} + i_n = 0$

Let denote equivalent resistance as R_{eq}*!*

By KCL for node v_p *gives,* $\frac{v_p - v_2}{20000} + \frac{v_p}{40000} + i_p = 0$

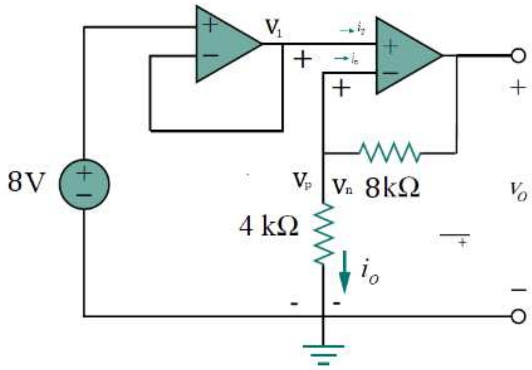
By substitution, $v_{output} = 2(v_2 - v_1) = 0.02V$

This implies that, $i_0 = \frac{v_{output}}{1000} = 2 \times 10^{-5}A = 20\mu A$

5.9

Let redraw the amplifier with some important voltages and currents!

CONT...



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0$*!*

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n$*!*

It's clear that, $v_1 = 8V$

This implies, $v_p = v_n = 8V$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

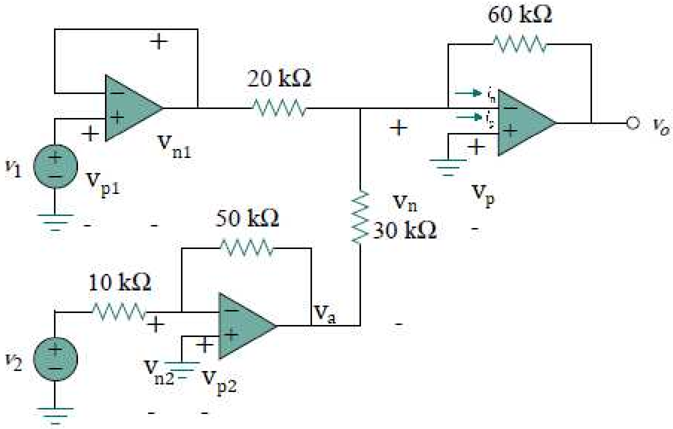
By KCL for node v_n *gives,* $\frac{8}{4000} + \frac{8 - v_0}{8000} + i_n = 0$

Rearranging the equation gives, $v_0 = \mathbf{24V}$

It's obvious that, $i_0 = \frac{v_n}{4000} = 2 \times 10^{-3} A = \mathbf{2mA}$

5.10

Let redraw the amplifier with some important voltages and currents!



It's obvious that, $v_{p1} - v_{n1} = 0 \Rightarrow v_{p1} = v_{n1}!$

This implies, $v_{p1} = v_{n1} = v_1$

Likewise, $v_{p2} - v_{n2} = 0 \Rightarrow v_{p2} = v_{n2}!$

This implies, $v_{p2} = v_{n2} = 0$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_{n2} *gives,* $\frac{v_{n2}-v_2}{10000} + \frac{v_{n2}-v_a}{50000} = 0$

Rearranging the equation gives, $v_a = -5v_2!$

By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

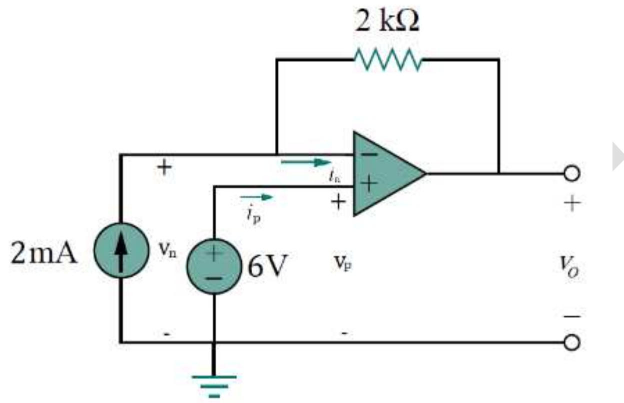
By KCL for node v_n *gives,* $\frac{v_n - v_{n1}}{20000} + \frac{v_n - v_a}{30000} + \frac{v_n - v_0}{60000} + i_n = 0$

Rearranging the equation gives, $v_0 = -3v_{n1} - 2v_a = -3v_1 + 10v_2$

By substitution, $v_0 = \mathbf{18V}$

5.9

(a) Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n!$

It's clear that, $v_p = 6V$

This implies, $v_p = v_n = 6V$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

CONT...

As it known, current flows from a higher potential to lower!

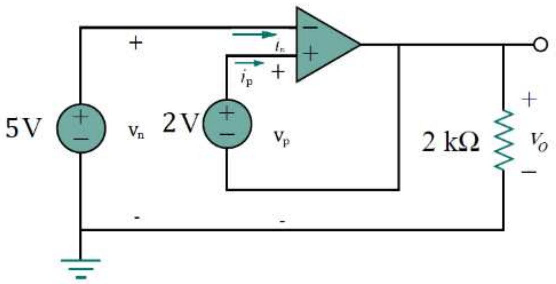
This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_n *gives,* $-0.002 + \frac{v_n - v_0}{2000} + i_n = \mathbf{0}$

Rearranging the equation gives, $v_0 = -4 + v_n = \mathbf{2V}$

(b) Let redraw the op amp with some important voltages and currents!



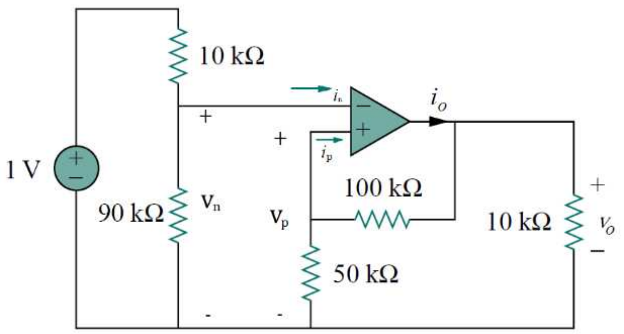
It's clear that, $v_n = \mathbf{5V}$

This implies, $v_p = v_n = \mathbf{5V}$

It's clear that, $v_p - v_0 = 2 \Rightarrow v_0 = \mathbf{3V}$

5.13

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0$*!*

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n$*!*

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections of resistors!*

CONT...

It's obvious that, $10k\Omega$ *and* $90k\Omega$ *are connected in series so* R_{eq} *for them is* $10k\Omega + 90k\Omega = 100k\Omega$

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

By substitution, $v_{90k\Omega} = v_n = v_p = \frac{90}{100} 1V = 0.9V$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_p *gives,* $\frac{v_p}{50000} + \frac{v_p - v_0}{100000} + i_p = 0$

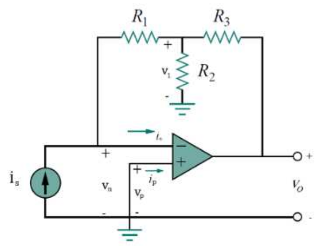
This implies, $v_0 = 3v_p = \mathbf{2.7V}$

By KCL for node i_0 *gives,* $-i_0 + \frac{v_0 - v_p}{100000} + \frac{v_0}{10000} = \mathbf{0}$

This implies, $i_0 = \frac{11v_0 - v_P}{100000} = 288 \times 10^{-4} A = \mathbf{288\mu A}$

5.15

(a) Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_n *gives,* $-i_s + \frac{v_n - v_1}{R_1} + i_n = 0$

By KCL for node v_1 *gives,* $\frac{v_1 - v_n}{R_1} + \frac{v_1}{R_2} + \frac{v_1 - v_0}{R_3} = 0$

CONT...

Rearranging the equations gives,

$i_s = -\frac{v_1}{R_1}$

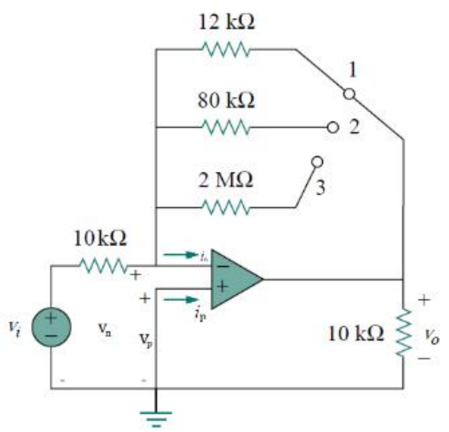
$v_0 = v_1\left(\frac{R_3}{R_1} + \frac{R_3}{R_2} + 1\right)$

This implies, $\boldsymbol{\frac{v_0}{i_s} = -R_1\left(\frac{R_3}{R_1} + \frac{R_3}{R_2} + 1\right) = -R_2 - \frac{R_3 R_1}{R_2} - R_1}$

(b) By substitution, $\frac{v_0}{i_s} = \boldsymbol{-92k\Omega}$

5.17

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

(a) By KCL for node v_n *gives,* $\frac{v_n - v_i}{10000} + \frac{v_n - v_0}{12000} + i_n = 0$

Rearranging the equation gives, $5v_0 = -6v_i$

This implies, $\frac{v_0}{v_i} = -1.2$

(b) By KCL for node v_n *gives,* $\frac{v_n - v_i}{10000} + \frac{v_n - v_0}{80000} + i_n = 0$

Rearranging the equation gives, $v_0 = -8v_i$

CONT . . .

This implies, $\frac{v_0}{v_i} = -8$

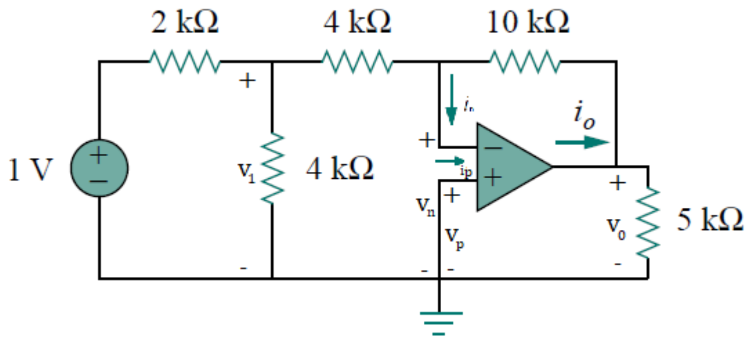
(c) By KCL for node v_n *gives,* $\frac{v_n - v_i}{10000} + \frac{v_n - v_0}{2000000} + i_n = 0$

Rearranging the equation gives, $v_0 = \boldsymbol{-200 v_i}$

This implies, $\frac{v_0}{v_i} = \boldsymbol{-200}$

5.19

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_1 *gives,* $\frac{v_1 - 1}{2000} + \frac{v_1}{4000} + \frac{v_1 - v_n}{4000} = 0 \Rightarrow v_1 = 0.5V$

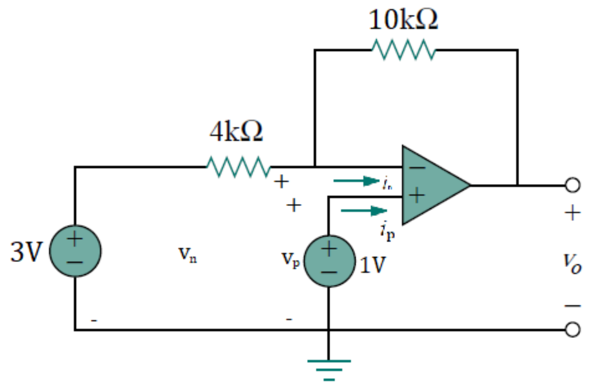
By KCL for node v_n *gives,* $\frac{v_n - v_1}{4000} + \frac{v_n - v_0}{10000} + i_n = 0 \Rightarrow v_0 = -1.25V$

By KCL for node v_0 *gives,* $\frac{v_0 - v_n}{10000} - i_0 + \frac{v_0}{5000} = 0 \Rightarrow i_0 = \frac{3v_0}{10000} = -3.75 \times 10^{-4} A = -3.75\mu A$

5.21

Let redraw the op amp with some important voltages and currents!

CONT...



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0$*!*

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 1V$*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

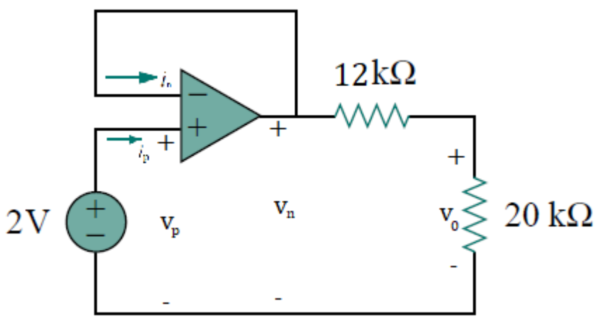
Let denote currents leaving a node positive!

By KCL for node v_n *gives,* $\frac{v_n - 3}{4000} + \frac{v_n - v_0}{10000} + i_n = \mathbf{0}$

Rearranging the equation gives, $v_0 = \frac{7v_n - 15}{2} = \boldsymbol{-4V}$

5.25

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0$*!*

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 2V$*!*

CONT . . .

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections of resistors!*

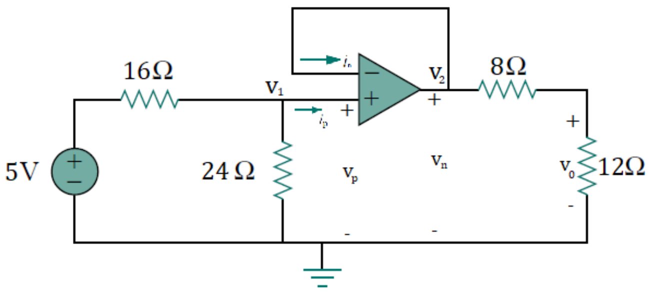
It's obvious that, $12k\Omega$ *and* $20k\Omega$ *are connected in series so* R_{eq} *for them is* $12k\Omega + 20k\Omega = 32k\Omega$

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

By substitution, $v_0 = \frac{20}{32} 2V = \mathbf{1.25V}$

5.27

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0$*!*

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n$*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections of resistors!*

It's obvious that, 16Ω *and* 24Ω *are connected in series so* R_{eq} *for them is* $16\Omega + 24\Omega = 40\Omega$

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

By substitution, $v_1 = \frac{24}{40} 5V = 3V$

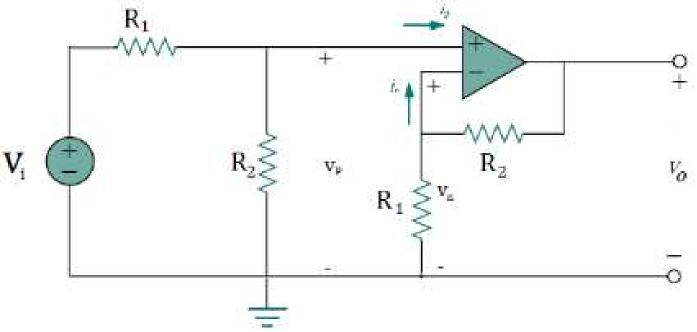
This implies, $v_1 = v_p = v_n = v_2 = 3V$*!*

Likewise, 8Ω *and* 12Ω *are connected in series so* R_{eq} *for them is* $8\Omega + 12\Omega = 20\Omega$

By substitution, $v_0 = \frac{12}{20} 3V = \mathbf{1.8V}$

5.29

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0$*!*

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n$*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections of resistors!*

It's obvious that, R_1 *and* R_2 *are connected in series so* R_{eq} *for them is* $R_1 + R_2$*!*

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

By substitution, $v_{R_2} = v_p = v_n = \frac{R_2}{R_1+R_2} v_i$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_n *gives,* $\frac{v_n}{R_1} + \frac{v_n - v_0}{R_2} + i_n = 0$

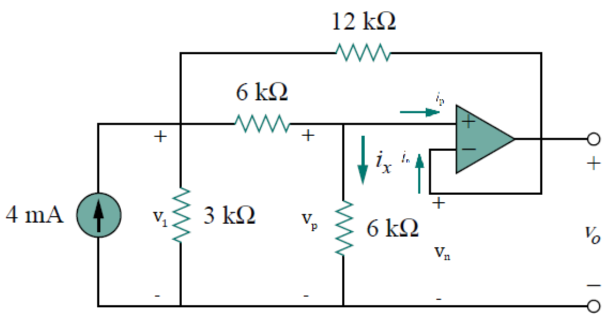
Rearranging the equations gives, $v_0 = v_n \left(\frac{R_2}{R_1} + 1\right)$*!*

By substitution, $v_0 = v_i \cdot \left(\frac{R_1}{R_1+R_2}\right) \cdot \left(\frac{R_2}{R_1} + 1\right)$

This implies, the voltage gain is $\boldsymbol{\frac{v_0}{v_i} = \frac{R_2}{R_1}}$***!***

5.31

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0$*!*

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = v_0$*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_1 *gives,* $-0.004 + \frac{v_1}{3000} + \frac{v_1 - v_p}{6000} + \frac{v_1 - v_0}{12000} = 0$

By KCL for node v_p *gives,* $\frac{v_p - v_1}{6000} + \frac{v_p}{6000} + i_p = 0$

Rearranging the equations gives,

$$v_1\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{12}\right) + v_0\left(-\frac{1}{6} - \frac{1}{12}\right) = 4$$

$$v_1\left(-\frac{1}{6}\right) + v_0\left(\frac{1}{6} + \frac{1}{6}\right) = 0$$

As it known, $\boldsymbol{Ax} = \boldsymbol{b} \Rightarrow \boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b} \Rightarrow \boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$

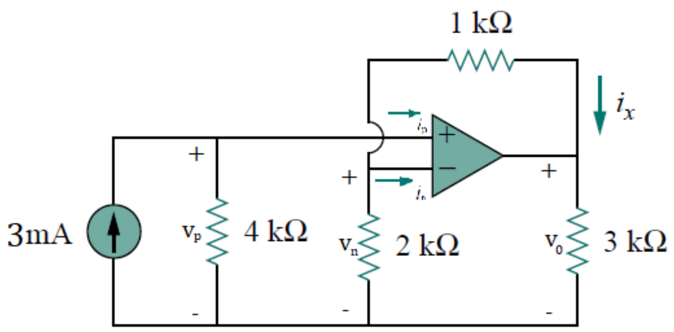
$$\begin{bmatrix} \frac{7}{12} & -\frac{1}{4} \\ -\frac{1}{6} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_0 \end{bmatrix} = \begin{bmatrix} 8.7273 \\ 4.3636 \end{bmatrix}$$

This implies, $v_1 = v_p = v_n = 8.7273V$ *and* $v_0 = v_p = v_n = 4.3636V$

It's clear that, $i_x = \frac{v_p}{6000} = 7.2727 \times 10^{-3}A = \boldsymbol{7.2727mA}$

5.33

Let redraw the op amp with some important voltages and currents!



By the definition of ideal op amp, $i_n = i_p = 0$*!*

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n$*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_p *gives,* $-0.003 + \frac{v_p}{4000} + i_p = 0$

This implies, $v_p = v_n = 12V$*!*

By KCL for node v_n *gives,* $\frac{v_n}{2000} + \frac{v_n - v_0}{1000} + i_n = 0$

This implies, $v_0 = 1.5v_n = 1.5v_p = 18V$

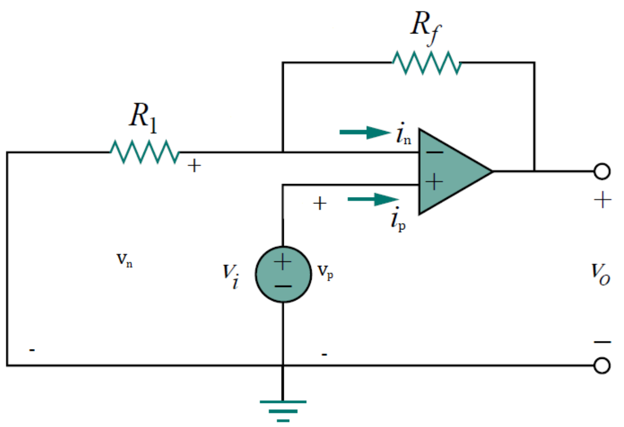
It's clear that, $i_x = \frac{v_n - v_0}{1000} = -6 \times 10^{-3}A = \boldsymbol{-6mA}$

By the definition of power, $p = \frac{dw(q)}{dt} = \frac{dw(q)}{dq} \cdot \frac{dq(t)}{dt} = v \cdot i = \frac{v^2}{R}$

This implies that, $p_{v_0} = \frac{v_0^2}{3000} = 0.108W = \boldsymbol{108mW}$

5.35

Let draw a basic non-inverting op amp circuit with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0$*!*

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = v_i$*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_n *gives,* $\frac{v_n}{R_1} + \frac{v_n - v_0}{R_f} + i_n = 0$

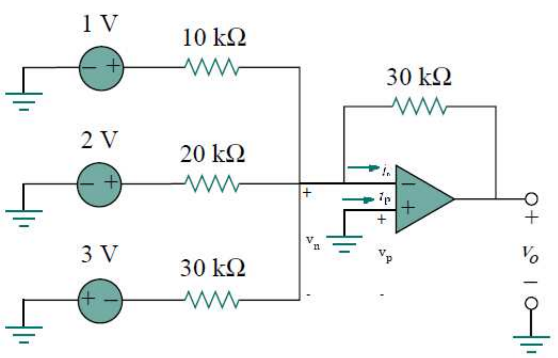
This implies that, $v_0 = \left(1 + \frac{R_f}{R_1}\right) v_i$

So that, $1 + \frac{R_f}{R_1} = 10 \Rightarrow \frac{R_f}{R_1} = \mathbf{9}$

By using APENDIX, if $R_f = 90k\Omega$ *then,* $R_1 = \mathbf{10k\Omega}$*!*

5.37

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

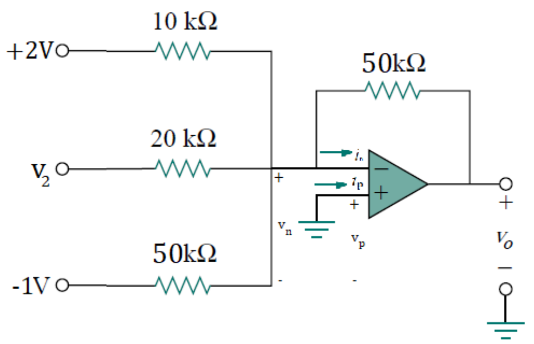
This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_n *gives,* $\frac{v_n - 1}{10000} + \frac{v_n - 2}{20000} + \frac{v_n + 3}{30000} + \frac{v_n - v_0}{30000} + i_n = 0 \Rightarrow v_0 = \mathbf{-3V}$

5.39

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

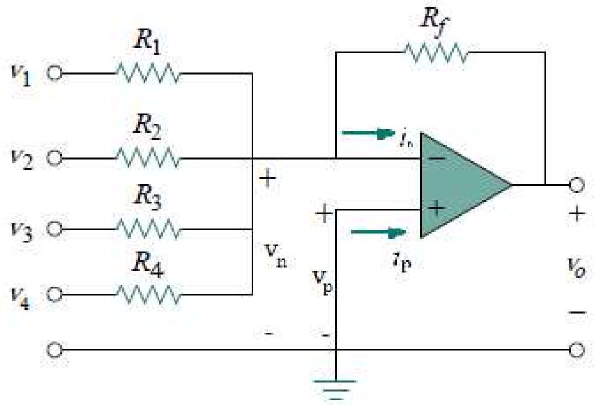
This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_n *gives,* $\frac{v_n - 2}{10000} + \frac{v_n - v_2}{20000} + \frac{v_n + 1}{50000} + \frac{v_n - v_0}{50000} + i_n = \mathbf{0} \Rightarrow v_2 = \frac{-2v_0 - 18}{5} \Rightarrow v_2 = \mathbf{3v}$

5.41

Let draw the basic summing op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_n *gives,* $\frac{v_n - v_1}{R_1} + \frac{v_n - v_2}{R_2} + \frac{v_n - v_3}{R_3} + \frac{v_n - v_4}{R_4} + \frac{v_n - v_0}{R_f} + i_n = 0$

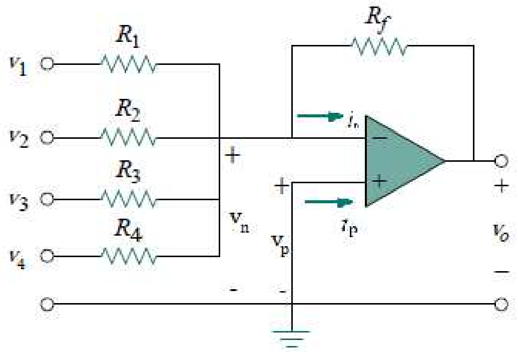
$\Rightarrow v_0 = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{Rf}{R_3} v_3 + \frac{Rf}{R_4} v_4\right)$

So that, $\frac{R_f}{R_1} = \frac{R_f}{R_2} = \frac{R_f}{R_3} = \frac{R_f}{R_4} = \frac{1}{4}$

By using APENDIX, if $R_f = 10k\Omega$ *then,* $R_1 = R_2 = R_3 = R_4 = \mathbf{40k\Omega}$

5.43

Let draw the basic summing op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_n *gives,* $\frac{v_n - v_1}{R_1} + \frac{v_n - v_2}{R_2} + \frac{v_n - v_3}{R_3} + \frac{v_n - v_4}{R_4} + \frac{v_n - v_0}{R_f} + i_n = 0$

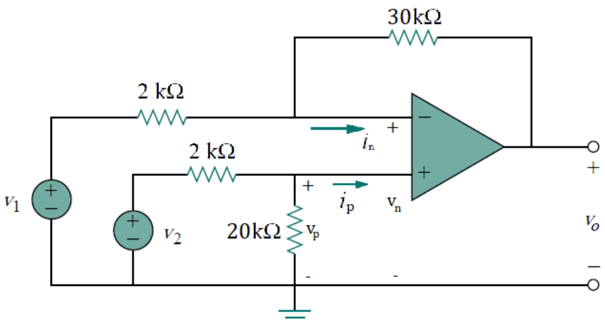
$\Rightarrow v_0 = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{Rf}{R_3} v_3 + \frac{Rf}{R_4} v_4\right)$

To create an averaging amplifier, the following equation must be satisfied $\boldsymbol{\frac{R_f}{R_1} = \frac{R_f}{R_2} = \frac{R_f}{R_3} = \frac{R_f}{R_4} = \frac{1}{4}}!$

As it known, $R_1 = R_2 = R_3 = R_4 = \boldsymbol{4k\Omega} \Rightarrow R_f = \boldsymbol{3k\Omega}$

5.47

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n!$

Let denote equivalent resistance as $R_{eq}!$

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections of resistors!*

It's obvious that, $2k\Omega$ *and* $20k\Omega$ *are connected in series so* R_{eq} *for them is* $2k\Omega + 20k\Omega = 22k\Omega$

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

CONT...

By substitution, $v_p = v_n = \frac{20}{22}v_2$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

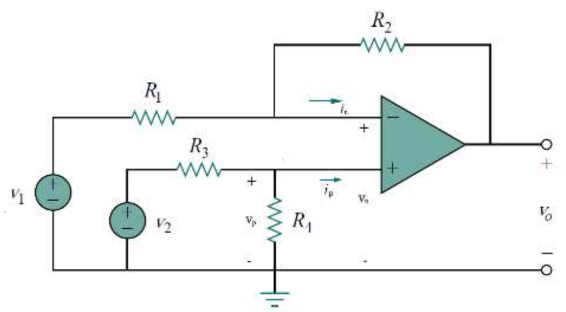
Let denote currents leaving a node positive!

By KCL for node v_n *gives,* $\frac{v_n - v_1}{2000} + \frac{v_n - v_0}{30000} + i_n = 0$

This implies, $v_0 = 16v_n - 15v_1 = 16\frac{20}{22}v_2 - 15v_1 = \mathbf{14.091V}$

5.49

Let draw the basic difference op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0$*!*

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n$*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_n *gives,* $\frac{v_n - v_1}{R_1} + \frac{v_n - v_0}{R_2} + i_n = 0$

Let denote equivalent resistance as R_{eq}*!*

As it known, $R_{eq} = \sum_{n=1}^{\infty} R_n$ *for the series connections of resistors!*

It's obvious that, R_3 *and* R_4 *are connected in series so* R_{eq} *for them is* $R_3 + R_4$*!*

By Voltage Division equation, $v_x = \frac{R_x}{R_{eq}} v_{source}$

CONT...

By substitution, $v_p = \frac{R_4}{R_3+R_4} v_2$

Substituting the equations gives, $v_0 = \frac{R_2(1+R_1/R_2)}{R_1(1+R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1$

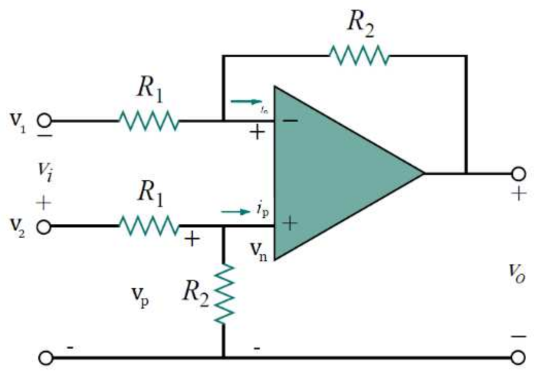
Since a difference amplifier must reject a signal common to the two inputs, the amplifier must have the property that $v_0 = 0$ *when* $v_1 = v_2$*!*

This implies, $\frac{\boldsymbol{R_1}}{\boldsymbol{R_2}} = \frac{\boldsymbol{R_3}}{\boldsymbol{R_4}} \Rightarrow \boldsymbol{v_0} = \frac{\boldsymbol{R_2}}{\boldsymbol{R_1}}(\boldsymbol{v_2} - \boldsymbol{v_1})$

So that, if $R_1 = R_3 = 10k\Omega$ *then* $R_2 = R_4 = \mathbf{20k\Omega}$*!*

5.53

(a) Let redraw the op amp with some important voltages and currents!



(a)

Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0$*!*

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n$*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_n *gives,* $\frac{v_n - v_1}{R_1} + \frac{v_n - v_0}{R_2} + i_n = 0$

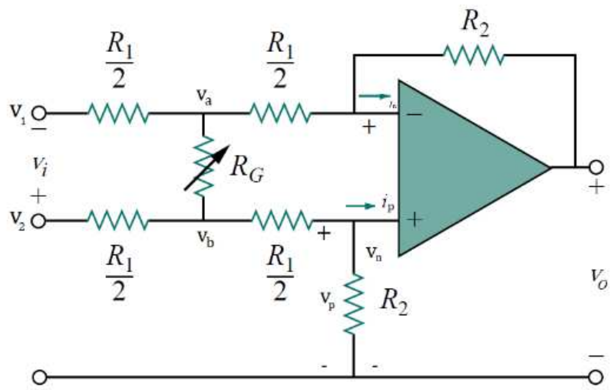
By KCL for node v_p *gives,* $\frac{v_p - v_2}{R_1} + \frac{v_p}{R_2} + i_p = 0$

By setting $v_p = v_n \Rightarrow \frac{R_2}{R_1+R_2} v_2 = \frac{R_2 v_1 + R_1 v_0}{R_1+R_2}$

It's clear that, $v_i = v_2 - v_1$*!*

This implies, $\frac{\boldsymbol{v_0}}{\boldsymbol{v_i}} = \frac{\boldsymbol{R_2}}{\boldsymbol{R_1}}$***!***

(b) Let redraw the op amp with some important voltages and currents!



(b)

Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0$*!*

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n$*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_a *gives,* $\frac{v_a - v_1}{R_1/2} + \frac{v_a - v_b}{R_G} + \frac{v_a - v_n}{R_1/2} = 0$

By KCL for node v_b *gives,* $\frac{v_b - v_2}{R_1/2} + \frac{v_b - v_a}{R_G} + \frac{v_b - v_p}{R_1/2} = 0$

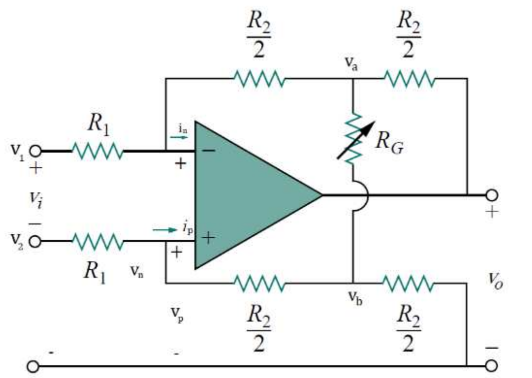
It's clear that, $v_p = v_n$*!*

This implies, $\frac{v_2 - v_1}{2} = \left(1 + \frac{R_1}{2R_6}\right)(v_b - v_a) = \frac{v_i}{2}$*!*

Equation for the difference amplifier gives, $v_0 = \frac{R_2}{R_1/2}(v_b - v_a)$

By using substitution, $\boldsymbol{\frac{v_0}{v_i} = \frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{1}{2R_G}}}$

(c) Let redraw the op amp with some important voltages and currents!



(c)

Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0$*!*

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n$*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_n *gives,* $\frac{v_n - v_1}{R_1} + \frac{v_n - v_a}{R_2/2} + i_n = 0$ *(1)*

By KCL for node v_p *gives,* $\frac{v_p - v_2}{R_1} + \frac{v_p - v_b}{R_2/2} + i_p = 0$ *(2)*

By KCL for node v_a *gives,* $\frac{v_a - v_n}{R_2/2} + \frac{v_a - v_b}{R_6} + \frac{v_A - v_0}{R_2/2} = 0$ *(3)*

By KCL for node v_b *gives,* $\frac{v_b - v_p}{R_2/2} + \frac{v_b - v_a}{R_G} + \frac{v_b}{R_2/2} = 0$ *(4)*

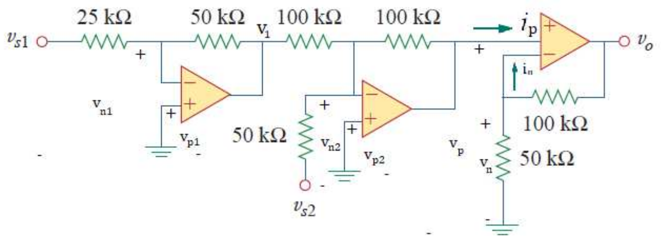
It's clear that, $\frac{v_2 - v_1}{2} = \left(1 + \frac{R_1}{2R_6}\right)(v_b - v_a) = \frac{v_i}{2}$ *(5)*

By subtracting the equation 4 from 3 gives, $-2(v_b - v_a) \cdot \left(1 + \frac{R_2}{2R_G}\right) = \boldsymbol{v_0}$

By substitution, $\boldsymbol{\frac{R_2}{R_1} v_i \left(1 + \frac{R_2}{2R_6}\right) = v_0 \Rightarrow \frac{v_0}{v_i} = \frac{R_2}{R_1}\left(1 + \frac{R_2}{2R_G}\right)}$

5.57

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_{p1} - v_{n1} = 0 \Rightarrow v_{p1} = v_{n1} = 0!$

Likewise, $v_{p2} - v_{n2} = 0 \Rightarrow v_{p2} = v_{n2} = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_{n1} *gives,* $\frac{v_{n1}-v_{s1}}{25000} + \frac{v_{n1}-v_1}{50000} = 0 \Rightarrow v_1 = -2v_{s1}$

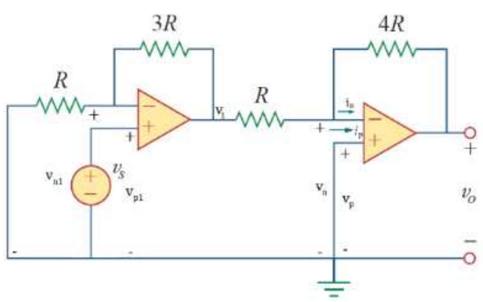
By KCL for node v_{n2} *gives,* $\frac{v_{n2}-v_1}{100000} + \frac{v_{n2}-v_{s2}}{50000} + \frac{v_{n2}-v_p}{100000} = 0 \Rightarrow v_p = v_n = -(v_1 + 2v_{s2})$

By KCL for node v_n *gives,* $\frac{v_n}{50000} + \frac{v_n - v_0}{100000} + i_n = 0 \Rightarrow v_0 = 3v_n$

By substitution, $\boldsymbol{v_0 = 3v_n = -3(v_1 + 2v_{s2}) = 6v_{s1} - 6v_{s2}!}$

5.59

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

It's obvious that, $v_{p1} - v_{n1} = 0 \Rightarrow v_{p1} = v_{n1} = v_s!$

CONT...

Likewise, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

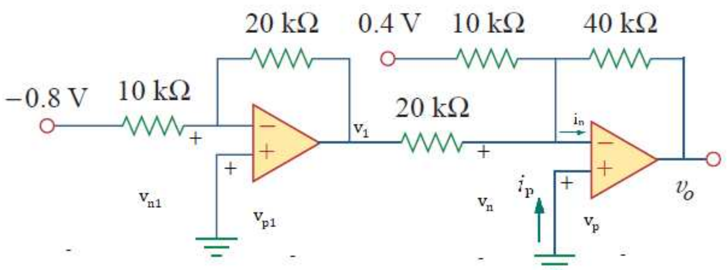
By KCL for node v_{n1} *gives,* $\frac{v_{n1}}{R} + \frac{v_{n1} - v_1}{3R} = 0 \Rightarrow v_1 = 4v_{n1}$

By KCL for node $\boldsymbol{v_n}$ ***gives,*** $\boldsymbol{\frac{v_n - v_1}{R} + \frac{v_n - v_0}{4R} + i_n = 0 \Rightarrow v_0 = -4v_1 = -16v_{n1} = -16v_s}$

This implies that, $\frac{v_0}{v_s} = \boldsymbol{-16}$

5.61

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_{p1} - v_{n1} = 0 \Rightarrow v_{p1} = v_{n1} = 0!$

Likewise, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

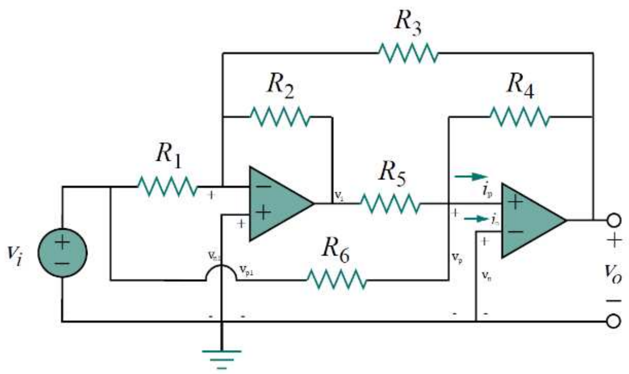
Let denote currents leaving a node positive!

By KCL for node v_{n1} *gives,* $\frac{v_{n1} + 0.8}{10000} + \frac{v_{n1} - v_1}{20000} = 0 \Rightarrow v_1 = \boldsymbol{1.6V}$

By KCL for node v_n *gives,* $\frac{v_n - 0.4}{10000} + \frac{v_n - v_1}{20000} + \frac{v_n - v_0}{40000} + i_n = 0 \Rightarrow v_0 = \boldsymbol{-4.8V}$

5.63

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

It's obvious that, $v_{p1} - v_{n1} = 0 \Rightarrow v_{p1} = v_{n1} = 0!$

Likewise, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

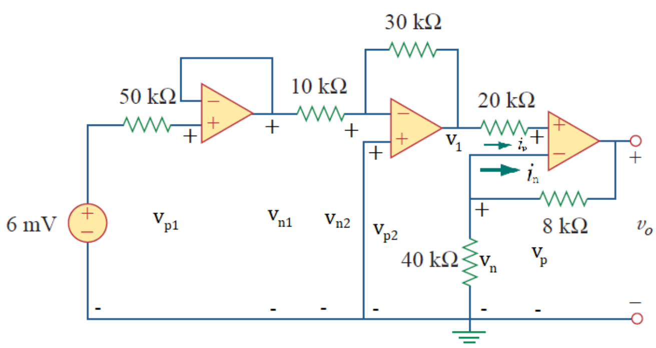
By KCL for node v_{n1} *gives,* $\frac{v_{n1}-v_i}{R_1} + \frac{v_{n1}-v_1}{R_2} + \frac{v_{n1}-v_0}{R_3} = 0 \Rightarrow v_1 = -\frac{R_2}{R_1}v_i - \frac{R_2}{R_3}v_0$

By KCL for node v_p *gives,* $\frac{v_p-v_1}{R_5} + \frac{v_p-v_i}{R_6} + \frac{v_p-v_0}{R_4} + i_p = 0 \Rightarrow v_0 = -\frac{R_4}{R_5}v_1 - \frac{R_4}{R_6}v_i$

By substitution, $\boldsymbol{v_0\left(1 - \frac{R_2R_4}{R_3R_5}\right) = v_i\left(\frac{R_2R_4}{R_1R_5} - \frac{R_4}{R_6}\right) \Rightarrow \frac{v_0}{v_i} = \frac{\frac{R_2R_4}{R_1R_5} - \frac{R_4}{R_6}}{1 - \frac{R_2R_4}{R_3R_5}}}$

5.65

Let redraw the op amp with some important voltages and currents!



CONT...

Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_{p1} - v_{n1} = 0 \Rightarrow v_{p1} = v_{n1} = 6mV!$

Likewise, $v_{p2} - v_{n2} = 0 \Rightarrow v_{p2} = v_{n2} = 0!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_{n2} *gives,* $\frac{v_{n2} - 0.006}{10000} + \frac{v_{n2} - v_1}{30000} = 0 \Rightarrow v_1 = -18mV$

By KCL for node v_n *gives,* $\frac{v_n}{40000} + \frac{v_n - v_0}{8000} + i_n = 0 \Rightarrow v_0 = 1.2v_n = 1.2v_p$

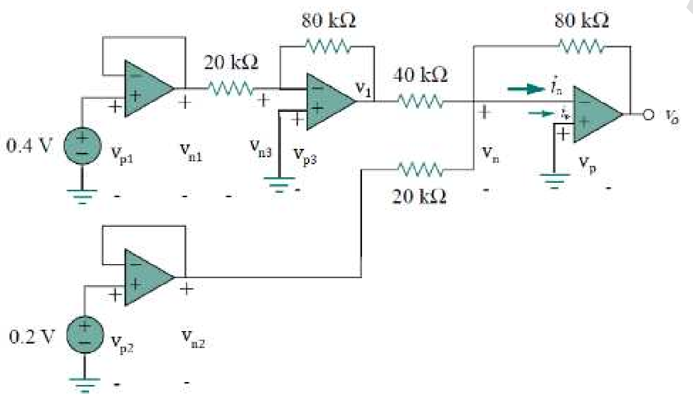
As it known, $i_n = i_p = 0!$

This implies that, the voltage drop across the $20k\Omega$ *resistor becomes zero!*

So that, $v_p = v_1 \Rightarrow v_0 = 1.2v_p = 1.2v_1 = \mathbf{-21.6mV}$

5.67

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_{p1} - v_{n1} = 0 \Rightarrow v_{p1} = v_{n1} = 0.4V!$

Likewise, $v_{p2} - v_{n2} = 0 \Rightarrow v_{p2} = v_{n2} = 0.2V!$

It's clear that, $v_{p3} - v_{n3} = 0 \Rightarrow v_{p3} = v_{n3} = 0!$

CONT...

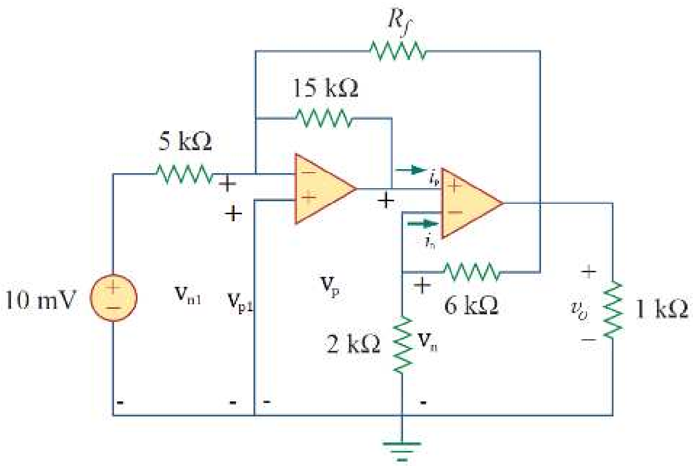
It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By KCL for node v_{n3} *gives,* $\frac{v_{n3}-v_{n1}}{20000} + \frac{v_{n3}-v_1}{80000} = 0 \Rightarrow v_1 = -4v_{n1} = \mathbf{-1.6V}$

By KCL for node v_n *gives,* $\frac{v_n-v_1}{40000} + \frac{v_n-v_{n2}}{20000} + \frac{v_n-v_0}{80000} = 0 \Rightarrow v_0 = -2v_1 - 4v_{n2} = \mathbf{2.4V}$

5.69

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

It's obvious that, $v_{p1} - v_{n1} = 0 \Rightarrow v_{p1} = v_{n1} = 0!$

Likewise, $v_p - v_n = 0 \Rightarrow v_p = v_n!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher}-v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_{n1} *gives,* $\frac{v_{n1}-0.010}{5000} + \frac{v_{n1}-v_p}{15000} + \frac{v_{n1}-v_0}{R_f} = 0 \Rightarrow v_0 = -R_f\left(\frac{0.030+v_p}{15000}\right)$

By KCL for node v_n *gives,* $\frac{v_n}{2000} + \frac{v_n-v_0}{6000} + i_n = 0 \Rightarrow v_0 = 4v_n = 4v_p$

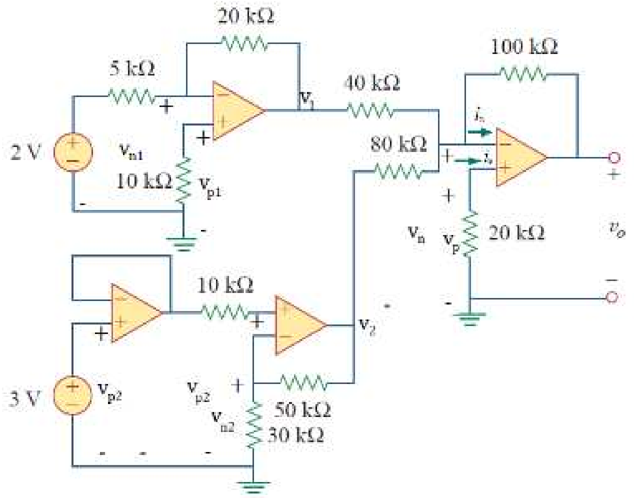
While $R_f = 10k\Omega, v_0 = -\left(\frac{0.30+10v_p}{15}\right)!$

By substitution, $4v_p = -\left(\frac{0.30+10v_p}{15}\right) \Rightarrow v_p = \mathbf{-4.2857mV}$

This implies, $v_0 = 4v_p = \mathbf{-17.143mV}$

5.71

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0!$

It's obvious that, $v_{p1} - v_{n1} = 0 \Rightarrow v_{p1} = v_{n1} = 0!$

Likewise, $v_{p2} - v_{n2} = 0 \Rightarrow v_{p2} = v_{n2} = 3V!$

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = 0!$

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_{n1} *gives,* $\frac{v_{n1}-2}{5000} + \frac{v_{n1}-v_1}{20000} = 0 \Rightarrow v_1 = -8V$

The voltage drop across the $10k\Omega$ *resistor becomes zero!*

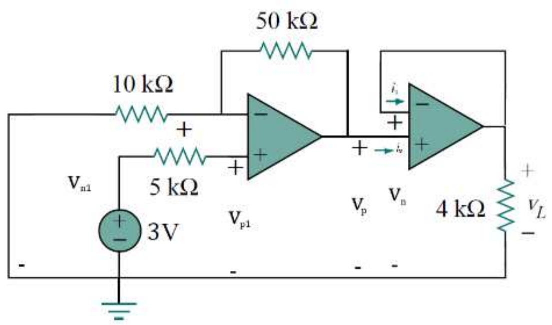
This implies, $v_{p2} = v_{22} = 3V!$

By KCL for node v_{p2} *gives,* $\frac{3}{30000} + \frac{3-v_2}{50000} = 0 \Rightarrow v_2 = \mathbf{8V}$

By KCL for node v_n *gives,* $\frac{v_n-v_1}{40000} + \frac{v_n-v_2}{80000} + \frac{v_n-v_0}{100000} + i_n = 0 \Rightarrow v_0 = \mathbf{-2.5v_1 - 1.25v_2 = 10V}$

5.73

Let redraw the op amp with some important voltages and currents!



Assume that the op amp is operating in its linear region!

By the definition of ideal op amp, $i_n = i_p = 0$*!*

It's obvious that, $v_{p1} - v_{n1} = 0 \Rightarrow v_{p1} = v_{n1} = 3V$*!*

It's obvious that, $v_p - v_n = 0 \Rightarrow v_p = v_n = v_0$*!*

By Ohm's Law, $v = i \cdot R \Rightarrow i = \frac{v}{R}$

As it known, current flows from a higher potential to lower!

This implies that, $i = \frac{v_{higher} - v_{lower}}{R}$

Let denote currents leaving a node positive!

By KCL for node v_{n1} *gives,* $\frac{v_{n1}}{10000} + \frac{v_{n1} - v_p}{50000} = 0 \Rightarrow v_p = v_n = v_0 = 6v_{n1} = \mathbf{18V}$

END!